How to Break Excel

Tom Button

 @tombutton

Binary fractions

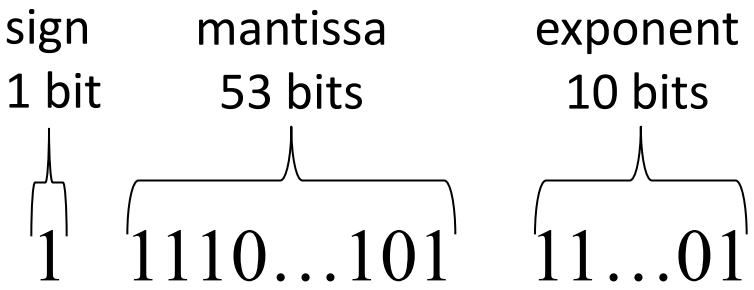
$$0.625_{10} = 0.101_2$$

2^0	2^{-1}	2^{-2}	2^{-3}
0	1	0	1

$$0.1_{10} = 0.000110011\ldots_2$$

$$0.9_{10} = 0.111001100\ldots_2$$

Storing numbers in Excel



$$0.1_{10} \approx 0.0001100110011001100110011001100110011001100110011001101_2$$

$$0.9_{10} \approx 0.1110011001100110011001100110011001100110011001100110011001101_2$$

$$0.1 \approx \frac{3602879701896397}{36028797018963968}$$

Where are the errors?

1

0.9

0.8

0.7 ↩ Error in the 53rd binary place

0.6

0.5

0.4

0.3 ↩ Error in the 55th binary place

0.2

0.1

1.38778×10^{-16}

$$2^{-53}+2^{-55}$$

$$=1.38788\times10^{-16}$$

Binary fractions: even denominators

$$\frac{2}{3} = 0.10101010\ldots_2$$

In Excel

$$\frac{2}{3} \approx 0.10101010101010101010101010101010101010101010101010101_2$$

$$\frac{2}{3} \approx \frac{6004799503160661}{9007199254740992}$$

2^{53}

But …

Excel **will** calculate $a^{\frac{1}{q}}$

where $a < 0,\ q \in \mathbb{N},\ q$ odd

Excel **won't** calculate $a^{\frac{p}{q}}$

where $a < 0,\ p, q \in \mathbb{N},\ p > 1,\ q$ odd

Thank you

 @tombutton