The Samaritani Formula

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(I help them commit it)

(I don't work for the University of Pisa. There is no Fraud Department there that I know of. Both of these are part of the point of the fraud.)

But before we start

1: Waitrose seems to have mostly fixed its negative pricing bug as of March this year but their fix has perhaps had unintended consequences. Also, I recently encountered a possible mistake in the fix.

2: Earlier this year I was able to add France, Spain and Portugal to my list of countries where mixed numbers don't exist. Italy I already knew about. Does anyone know of any more?

But before we start

For which positive real a does:

$$a^{a^{a^{\cdots}}}$$

make sense?

A disclaimer

I do not wish to suggest that what follows represents mainstream thought in Italy. Most Italians have probably never heard of Samaritani or his formula, and it seems to be obscure enough that most Italian mathematicians have not come across it even as a thing that needs debunking.

An analogy

Q: What does “socialised medicine” mean?

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A: The person speaking is probably American.

Q: What does “the Samaritani formula” mean?

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A: There is a good chance the person speaking either doesn't know what “independent” means or is hoping that you do not.

Of course it's only a good chance, not a guarantee, in both cases.

I am, for example, not American.

The Italian Lottery – some background

Actually there are two!

The Italian Lottery

Lotto: 11 “wheels” each with 90 numbers.
5 numbers drawn from each wheel. Yes, 990 numbers of which 55 numbers are drawn.
Fixed odds. (Is this unusual?)

Impossibly succinct

Using J:

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1+5?11$90
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86 29 82 51 90
84 68 52 29 75
30 55 87 75 18
69 89 15 88 16
48 14 80 66 74
18 49 82 46 13
24 53 50 81 77
22 49  5 59 31
69 32 51 17 36
87 26 78  4 20
62 38 35 48  4
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The Italian Lottery

Superenalotto: 90 numbers.
6 numbers are drawn.
622 million to 1 chance of getting all six right. Take that, new embiggened UK lottery!
Jackpot style.

The Italian Lottery

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However, moot since 2009 when SEL moved to having its own draws. Also has some sort of bonus number now? Only learned of de-heterodyning and bonus ball a few days ago.

The Italian Lottery

Mostly talking about “Lotto” today.

With SEL usual advice about avoiding commonly picked numbers applies.

Normal lotto no longer has orphans problem but did until surprisingly recently. (i.e. when I first ran across Samaritani)

The Italian Lottery

Lotto: You can bet on things right down to single numbers coming out. A single number pays about 10 to 1. Bets on more numbers are mostly much worse value.

The Italian Lottery

Lotto has been around for over 100 years. All sorts of stuff has sprung up around it, including “La Smorfia”, a bunch of advice about what to bet on based on what you dream about. I'd say this was outside my remit. (Does anyone actually believe in this stuff?). In any event, cf the film “47 Morto che Parla”. Though Wikipedia claims that should be just dead person and 48 is dead person talking.

The Italian Lottery

One thing that seems quite big in Italy is lottery “systems” which are actually block design type things. Apparently some lottery players in Italy have invented the best known block designs of some sizes. Seems sort of futile.

The Italian Lottery

“Ritardo” (delay): number of draws a given number has been absent.

The Italian Lottery

“Ritardista” (“delay theorist”, maybe?): one who would have you believe he or she “studies” delays and will sell you advice, systems, software. The Samaritani Formula would appear to be a free sample. Some of these people have been studying the lottery since before I was born, arrogant upstart that I am, and how dare I etc. etc.

The Italian Lottery

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The Italian Lottery

What is the “theoretical maximum delay” supposed to mean?
Perhaps not what it sounds like.

People turn up on it.scienza.matematica and other newsgroups asking about this every few years.

We tell them “there isn't one” or “infinite”. But they want to be told, or tell us, that the answer is 220.

Mockery through participation

Some say “theoretical maximum” doesn't mean what you or I might imagine. But some say it really is an actual limit. In which case we have some fun by asking what happens if 5 numbers on the same wheel have delay 220 at the same time. Clearly more than 5 can't, so saying 220 is a limit has some very odd consequences. They claim we are making fun of them. Yes indeed. In rare cases you could win the lottery 17 times in a row if these people were right.

Why 220?

Well, it's

$$\log_{(17/18)} 1/300000$$

you see. Samaritani's formula says this is the maximum possible delay. Or so his modern followers claim

Why? Was Samaritani an idiot?

17/18 because that's the probability a number won't be drawn. 300000 because it's 50 (numbers drawn each time) times 6000 (the number of draws so far. Ish. Well, it was in the 90s).

50? But you said...

Yes it's 55 now. Samaritani was around in the 30s when there were only 10 wheels. Or maybe there were only 10 in the late 90s when people were telling me this stuff. But anyway.

So it's not a limit at all?

No it grows slowly as time passes. Or modern delay theorists seem to say it does. In any event, on short timescales it doesn't change much.

For a lottery that had always been as the Italian lottery currently is, Samaritani's formula would be

Theoretical max delay = $\log_{(17/18)}(1/55n)$
After n draws

So it's not a limit at all?

In any event, delay theorists say that since 220 is (in their universe) a limit, start betting on a number when its delay reaches, say, 160, increasing your bet by 11% each time you don't win. When numbers reach very high delays, people bankrupt themselves trying to keep this up. (“Delays” are reported as though they were news / a real thing, in Italy). It could of course be argued these people are not the sharpest tools in the box to start with.

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So what is the S.F. really?

It actually seems like a perfectly decent estimate of “What do you think the longest delay seen will be, approximately, after n draws of an Italian-style lottery?”

Various ways to check / reconstruct

1: Simulate an Italian lottery with a computer program

2: Treat a single number as a biased coin. Calculate the distribution of lengths of max runs of tails in n tosses. Consider the whole lottery as 990 independent coins each tossed n times (which it isn't) or as one coin tossed 990n times (which it also isn't).

Various ways to check / reconstruct

3: (and we think this is what Samaritani may have done) After every draw, write down the delays the numbers had before they were drawn. Consider the 55n numbers you have after n draws. Imagine they are 55n values taken from a geometric distribution. And ask yourself about how big the largest of the 55n numbers will be.

Various ways to check / reconstruct

For large enough n, the Samaritani formula and (various measures of central tendency) of the distributions obtained from the other methods are all about the same. One odd thing is that the standard deviation of the distribution of max delay doesn't vary (much) with n, and this is apparently a known result.

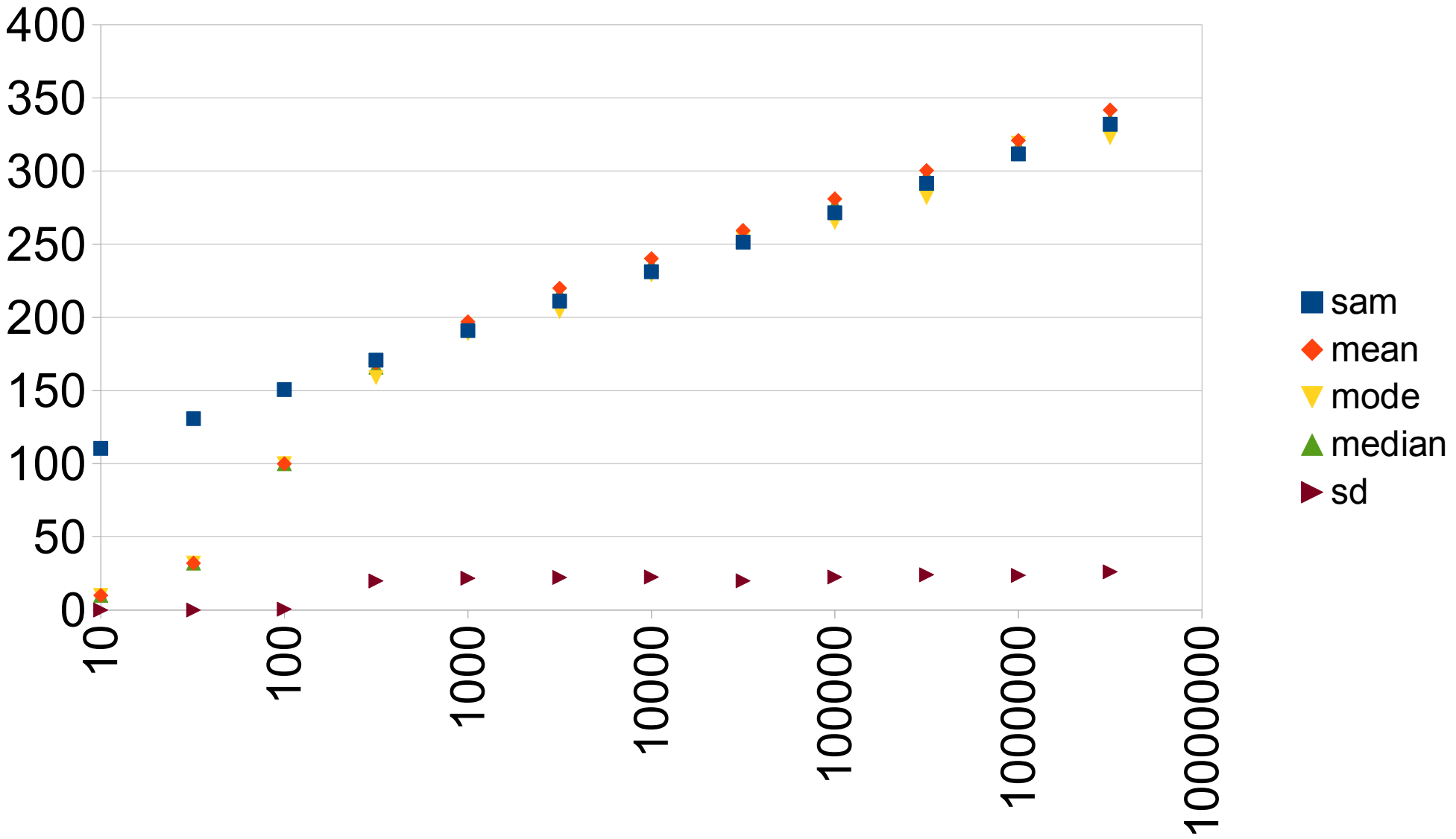
Various ways to check / reconstruct

Of course for very small n things go wrong. You can't treat the lottery like 990 independent coins as 55 numbers will definitely be drawn every time. For n up to 17, the maximum delay will definitely be n, and for n over 17 it cannot possibly be less than 17. For independent coins it could be 0.

Various ways to check / reconstruct

Treating the lottery as one coin tossed 990n times would allow delays longer than n, which is impossible. The “55n observations” method would ignore record breaking delays outstanding at the end of the run, and has the problem that the first 55 numbers are all definitely 0, the next 55 are at most 1, etc. Also if you use floating point arithmetic to calculate 990^{th} powers of cumulative distribution functions, arithmetic errors become a problem.

A graph of sim vs sam



Standard deviation

It is apparently a known result that the standard deviation of the longest run is roughly

$$1/\ln(1-p) \quad \text{or} \quad \sqrt{\pi^2/(6\ln^2(1/p)+1/12)}$$

The latter gives 22.4, which is about the value I get in simulations. Note that “n” is not present in the formula!

So what is the S.F. really?

This doesn't help you play the lottery, of course, unless someone somewhere says “I'm about to launch an Italian style lottery. Would you like to bet on what the longest delay will have been 100 years from now?” Even then it would seem preferable, these days, to use a computer simulation rather than the Samaritani Formula.

So, *was* Samaritani an idiot?

Our best guess is no. It seems much more likely that “delay theorists” have (perhaps wilfully) misinterpreted him, in which case he's a victim. One of these years I may see if I can track down a copy of his book.

Credits

When I first came across this way back when, I was part of a group including Elio Fabri and Francesco Romani in Pisa and Dani Ferrari in Rome who messed about with this. Dani Ferrari went to have a look at Samaritani's book in the national library in Rome. Look us and Samaritani up on Usenet for some of the original discussions. Some of it was on a private mailing list, however.