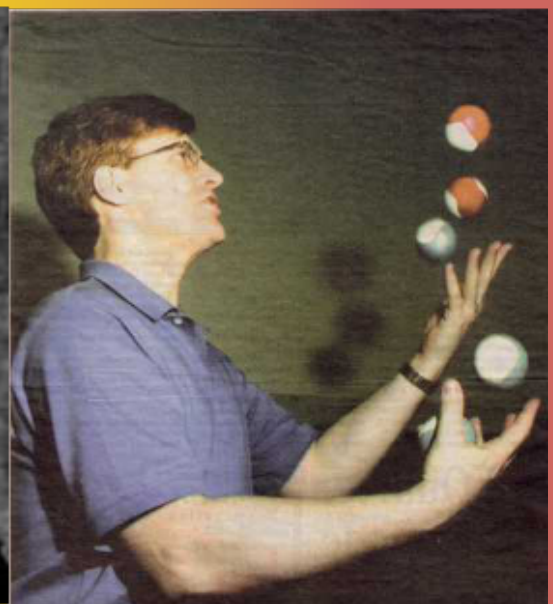


On Machin-like formulas and Gaussian integers

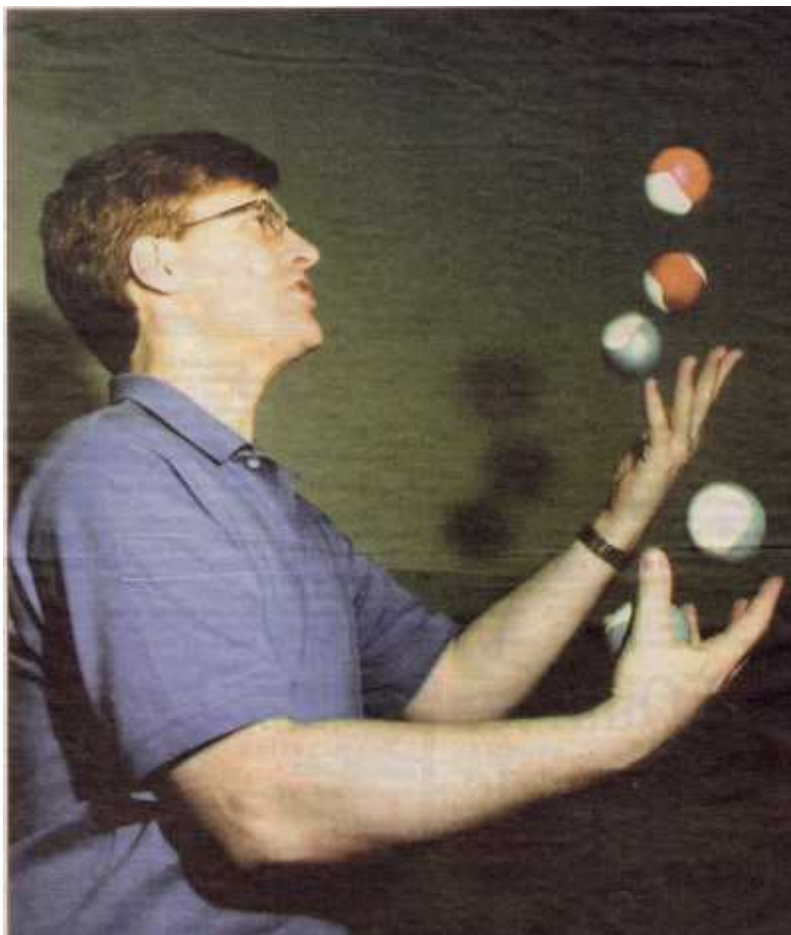
CARL FRIEDRICH GAUSS – JOHN MACHIN – COLIN WRIGHT

GAUSSIAN MACHIN-ATIONS

FULL PUZZLE

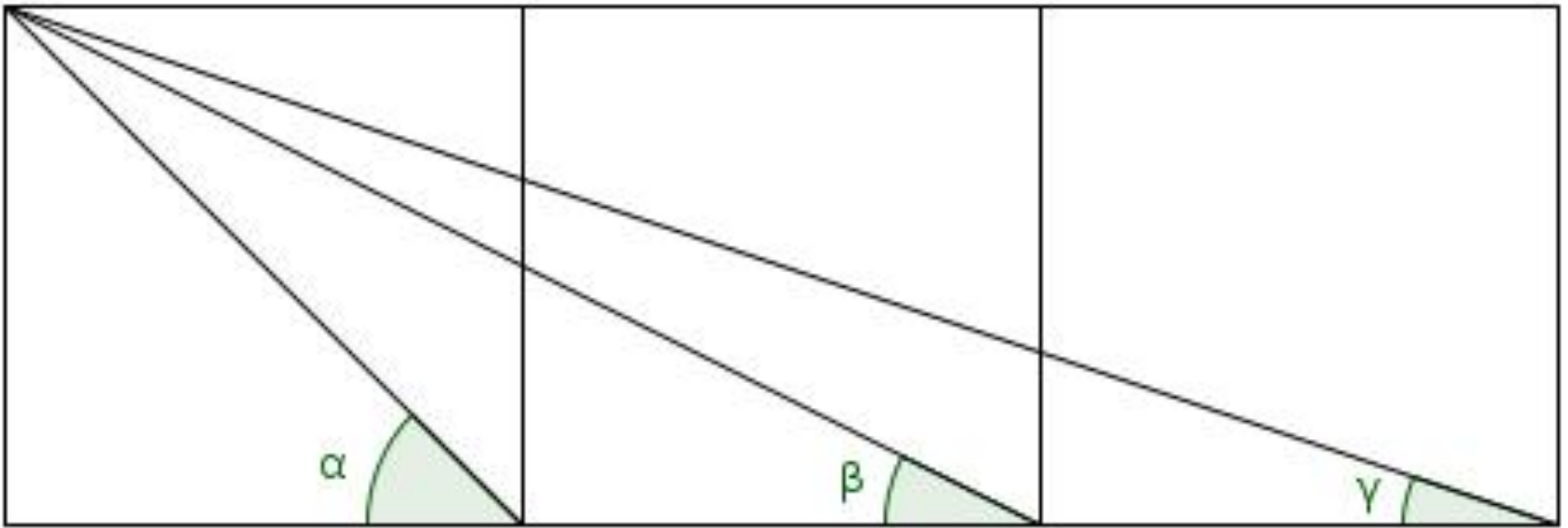


THE FIRST RULE OF MATHSJAM



**“DON’T TELL
THEM THE
ANSWER”
- COLIN WRIGHT**

SHOW THAT $\alpha = \beta + \gamma$



THE ATTACK OF THE GAUSSIAN INTEGERS

$$\gamma = \arg(3 + i)$$

$$\beta = \arg(2 + i)$$

$$\beta + \gamma = \arg((2 + i)(3 + i))$$

$$= \arg(5 + 5i)$$

$$= \frac{\pi}{4}$$

MACHIN'S FORMULA

$$4 \tan^{-1} \left(\frac{1}{5} \right) - \tan^{-1} \left(\frac{1}{239} \right) = \frac{\pi}{4}$$

$$(5 + i)^4 (239 - i) = 114,244 + 114,244i$$



JOHN MACHIN (1686-1751)

**USED THIS, AND THE TAYLOR
SERIES FOR ARCTAN, TO
COMPUTE π TO 100
DECIMAL PLACES (1706)**

GAUSS'S MACHIN-LIKE FORMULA

$$12 \tan^{-1}\left(\frac{1}{18}\right) + 8 \tan^{-1}\left(\frac{1}{57}\right) - 5 \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$$

$$(18 + i)^{12} (57 + i)^8 (239 - i)^5$$

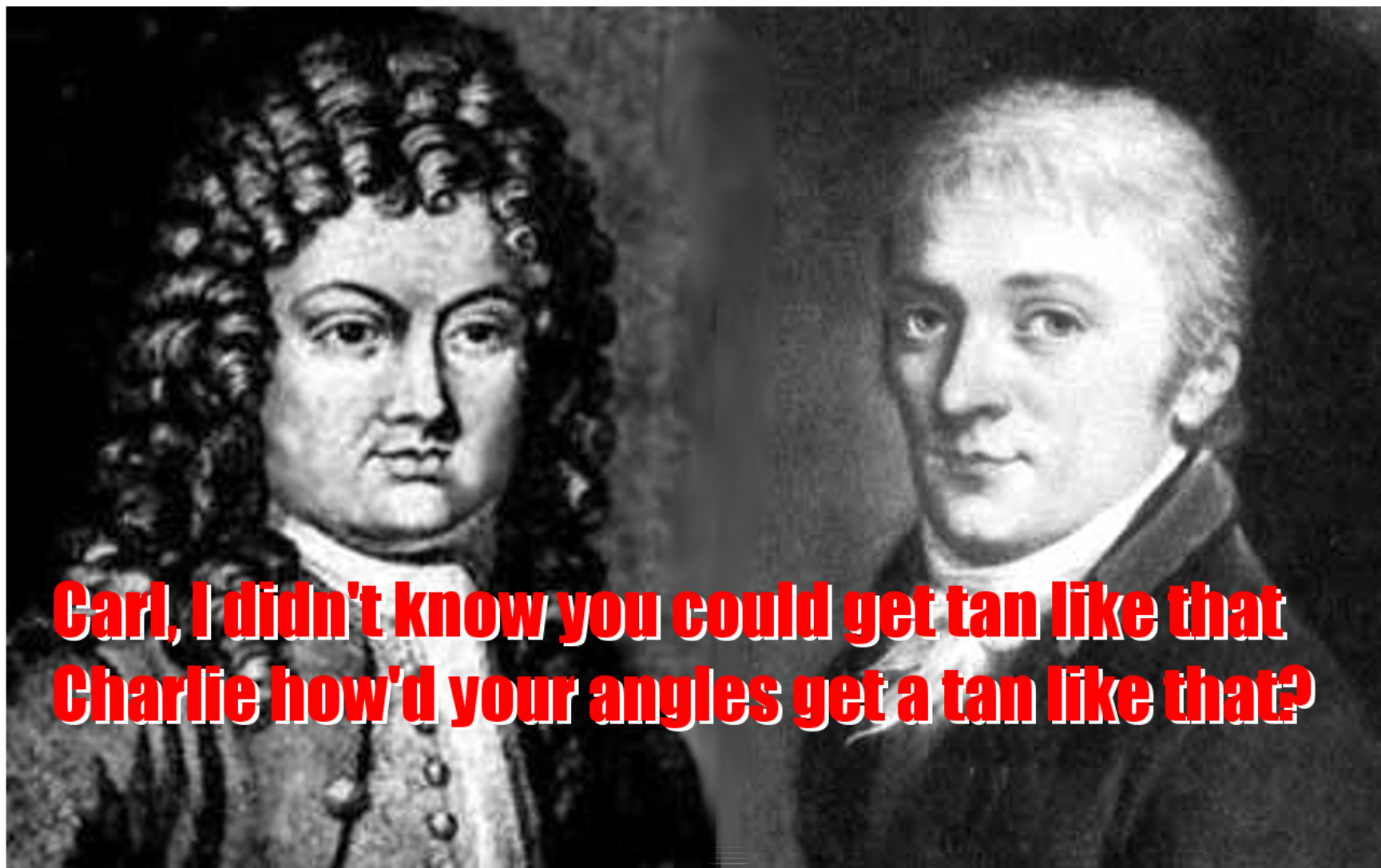
$$= 72,498,183,345,339,963,679,508,209,228,515,625,000,000 + \\ 72,498,183,345,339,963,679,508,209,228,515,625,000,000 i$$



**GAUSS'S FORMULA IS A
WHOLE 2% FASTER FOR
CALCULATING π .**

SUMMARY

- **THE 1-2-3 PUZZLE IS LINKED TO APPROXIMATIONS OF π**
- **YOU CAN CREATE MACHIN-LIKE FORMULAS BY FACTORISING GAUSSIAN INTEGERS**
- **WHAT WOULD MACHIN HAVE THOUGHT?**



**Carl, I didn't know you could get tan like that
Charlie how'd your angles get a tan like that?**