

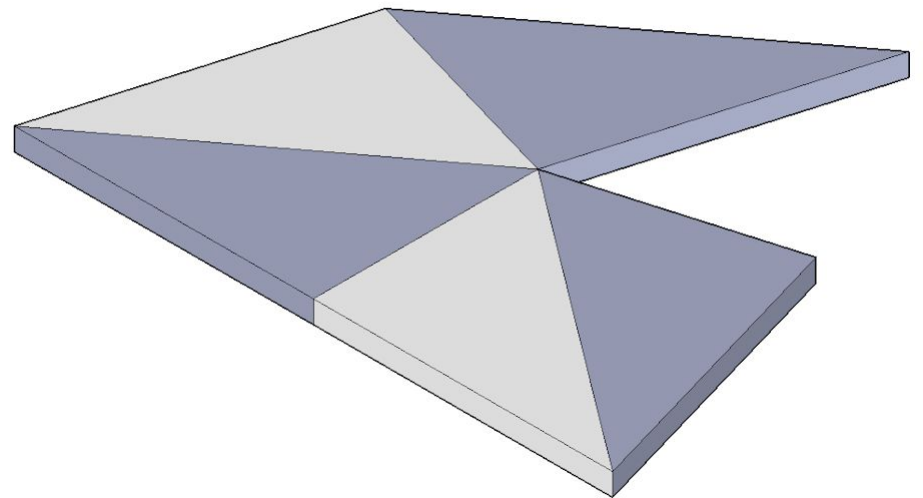
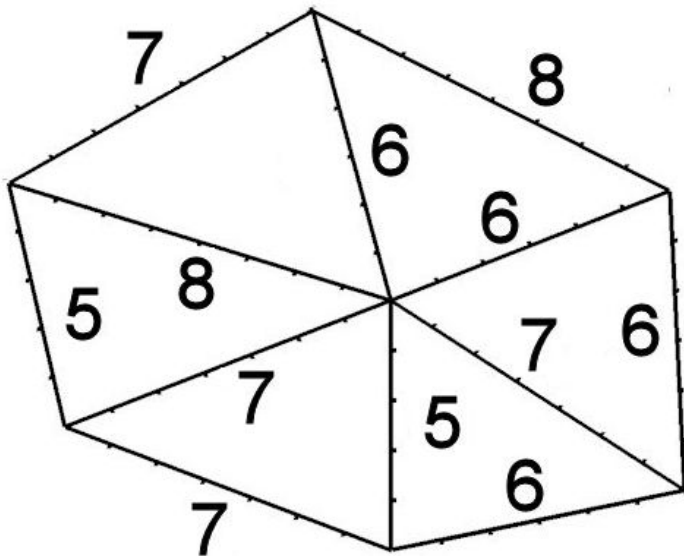
# MathsJam 2015

Donald Bell

"Curious and Interesting Triangles"

or

What makes a "nice" puzzle?

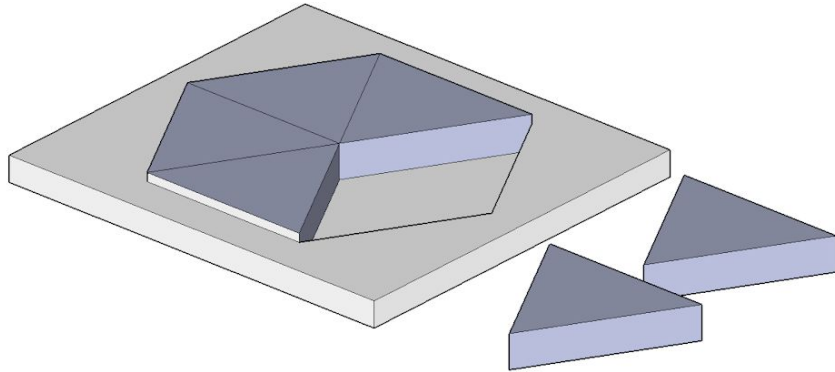


# The Characteristics of a "nice" Puzzle

- Challenging but not impossible (solution time 10-20 minutes)
- Not too hard (or expensive) to make
- Well presented (looks good on the coffee table)
- Contains a surprise (an "Aha!" moment)

Let's look at the "Hexasperation" and "3-4-5 Symmetry" puzzles

# Developing the "Hexasperation" puzzle



Version zero:  
Equilateral triangles in a  
regular hexagonal frame

Easy to make?

- yes, lots of similar pieces

Looks good on a coffee table?

- yes, in a well fitting frame

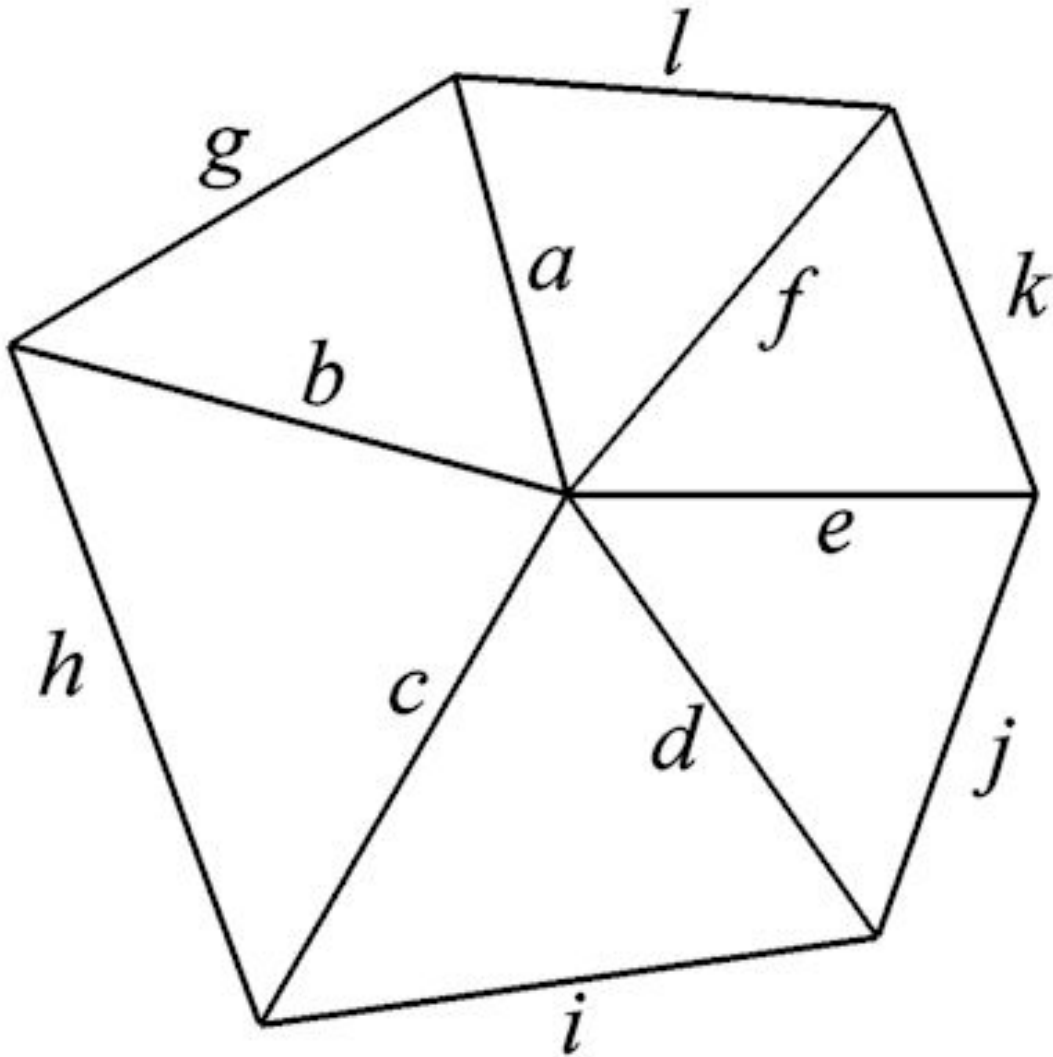
Challenging?

- well, no!

not even for a three-year old

(but it's a start)

# Developing the "Hexasperation" puzzle



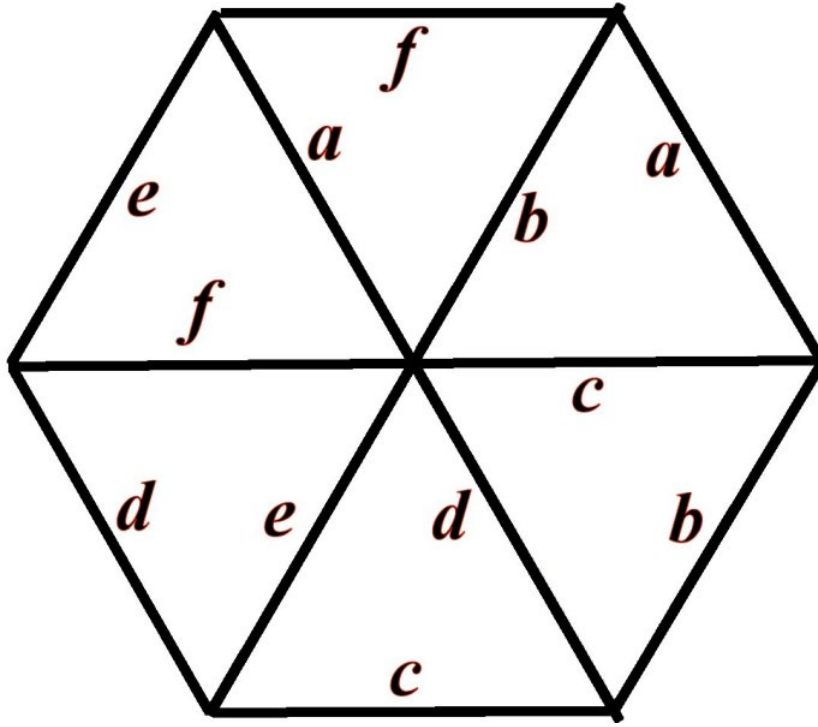
It is possible to have six different scalene triangles making up the hexagon.

That means up to 12 different lengths and 18 different angles.

But it is too easy to put the matching edges together.

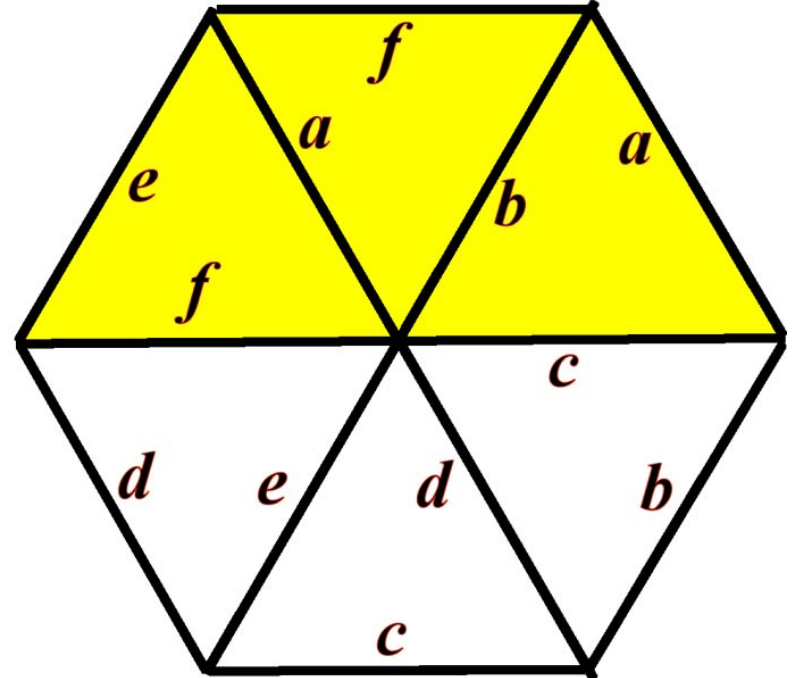
And it is much too difficult to make!

# Developing the "Hexasperation" puzzle

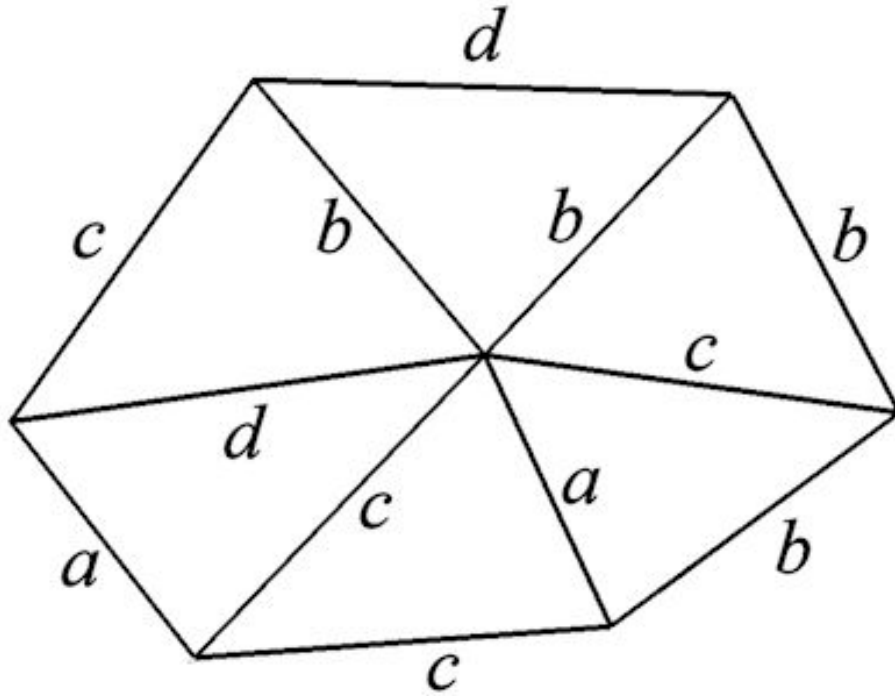


If the lengths of the "spokes" ( $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$ ) are made the same as the edges, then the puzzle begins to be a bit more challenging.

The three coloured triangles each have one edge of length  $a$ . So we don't immediately know which triangles should be touching.



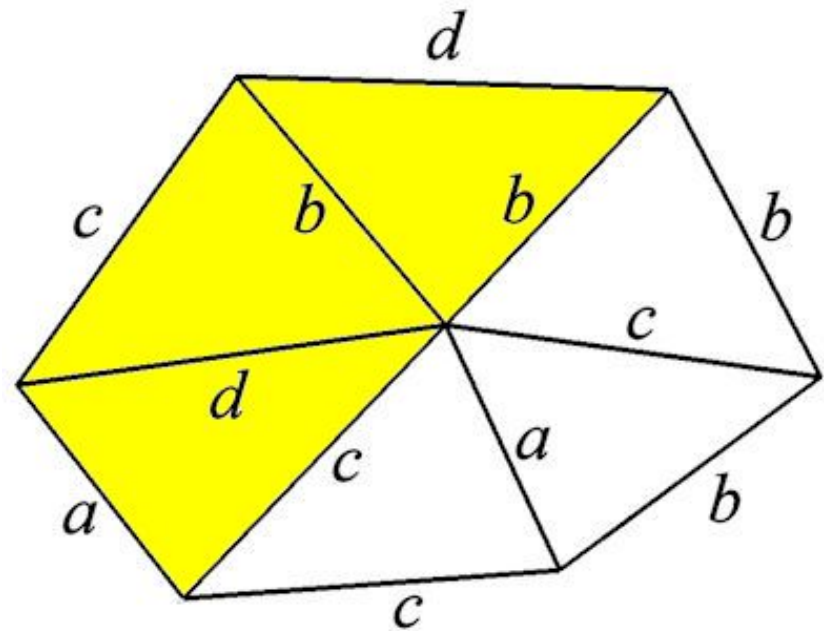
# Developing the "Hexasperation" puzzle



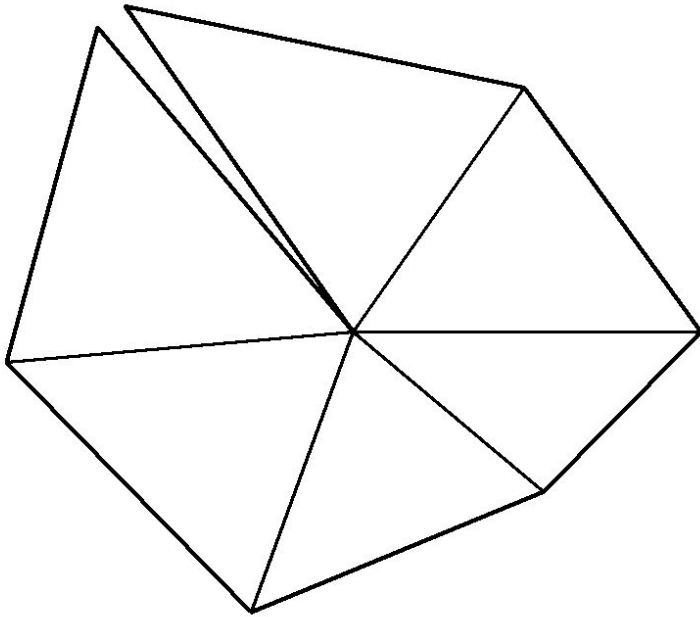
If the number of different lengths of the edges can be reduced to four, then the puzzle is a lot more challenging.

The three coloured triangles have one edge of length  $d$ .

And four of the triangles have at least one edge of length  $b$ .



# Developing the "Hexasperation" puzzle

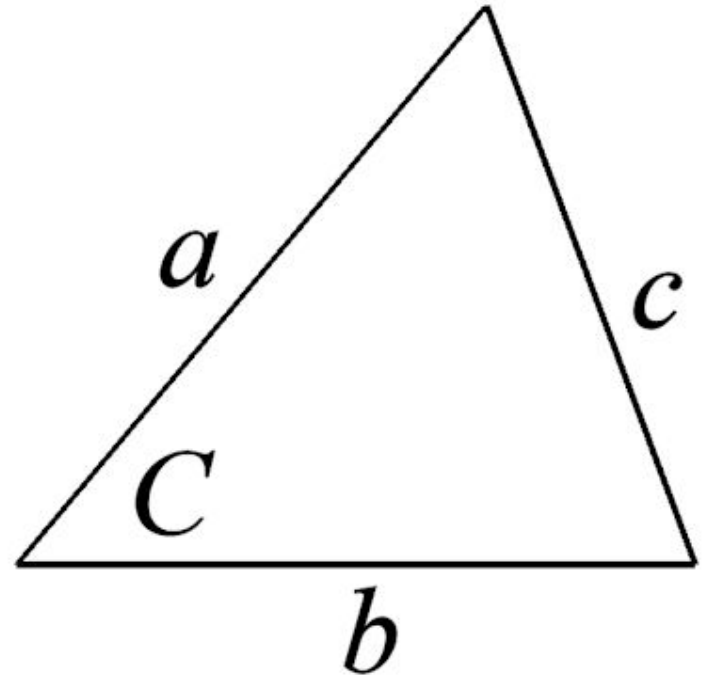


But unless the lengths of the triangles are chosen with some care, the central angles will not add up to 360 degrees

The cosine formula for calculating an angle of a triangle is:

$$\cos C = (a^2 + b^2 - c^2) / 2ab$$

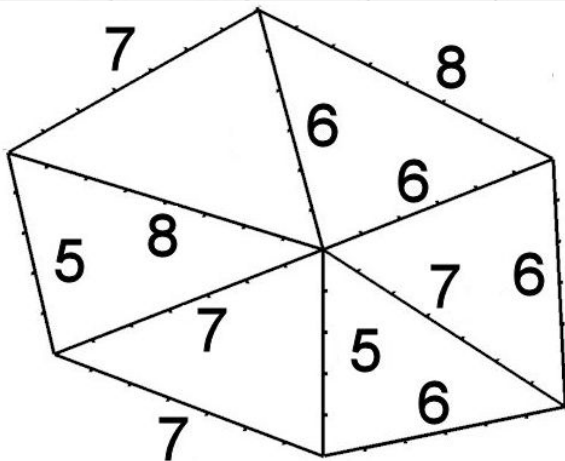
An Excel spreadsheet is needed to get the six triangle shapes right



# The "Hexasperation" puzzle spreadsheet

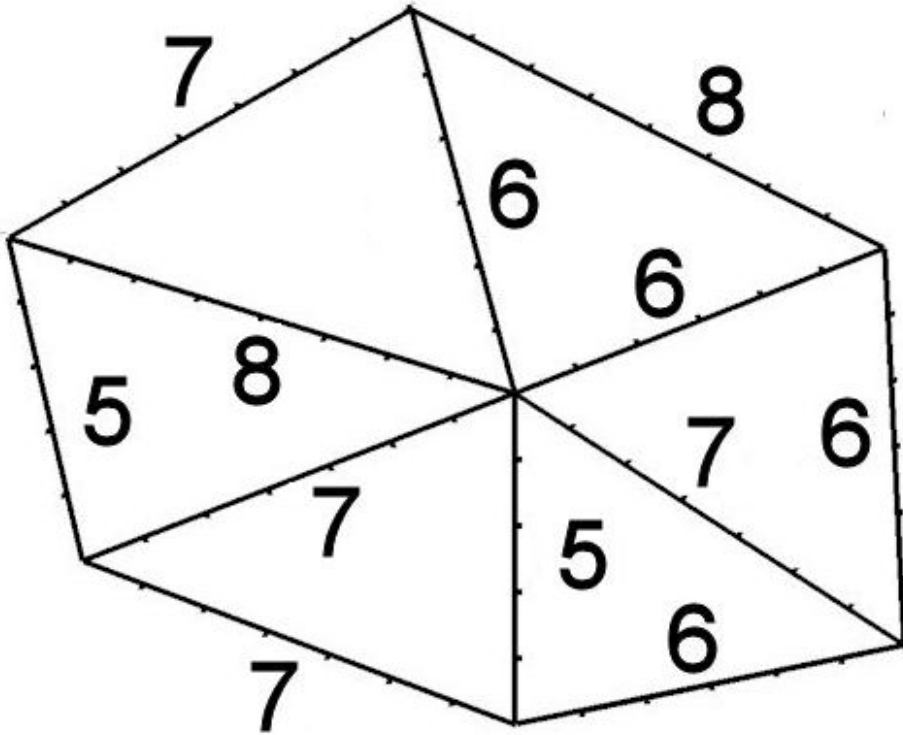
With some "trial and error" (and a spreadsheet to do the trigonometry), this is the final version of "Hexasperation".

	A	B	C	D	E	F	G	H	I
1	edge	A*A	B*B	C*C	A*A + B*B - C*C	2AB	cos(c)	angle c	degrees
2	5	25	49	36	38	70	0.5428571	0.996960874	57.12165044
3	7	49	36	36	49	84	0.5833333	0.947969741	54.31466529
4	6	36	36	64	8	72	0.1111111	1.459455312	83.62062979
5	6	36	64	49	51	96	0.53125	1.010721021	57.91004874
6	8	64	49	25	88	112	0.7857143	0.666946345	38.2132107
7	7	49	25	49	25	70	0.3571429	1.205589106	69.07516757
8	5	25	49						
9	7	49							
10									
11								TOTAL	360.2553725
12									



The sides are integers (5, 6, 7 and 8) and the angles at the centre add up to 360.26 degrees (which is a much smaller error than the errors in woodworking)

# The "Hexasperation" puzzle



The sides are integers  
(5, 6, 7 and 8)  
and the angles at the centre  
add up to 360.26

Evaluating the criteria for a good puzzle:

- Challenging? – certainly
- Easy to make? – well, three of the six triangles are isosceles
- Well presented? – no, a close fitting frame would make the puzzle too easy.
- A surprise or "AHa!" moment? – alas, no.

"this slide is deliberately blank"

It is covering up the next steps in the argument, because that would tell you too much about the final puzzle

# The "3-4-5 Symmetry" Puzzle

Surprisingly, the final puzzle has five triangles, not six, and it might, or might not, have anything to do with hexagons.

And all the triangles are identical.

But it DOES satisfy the four conditions of a "nice" puzzle!

# The "3-4-5 Symmetry" Puzzle

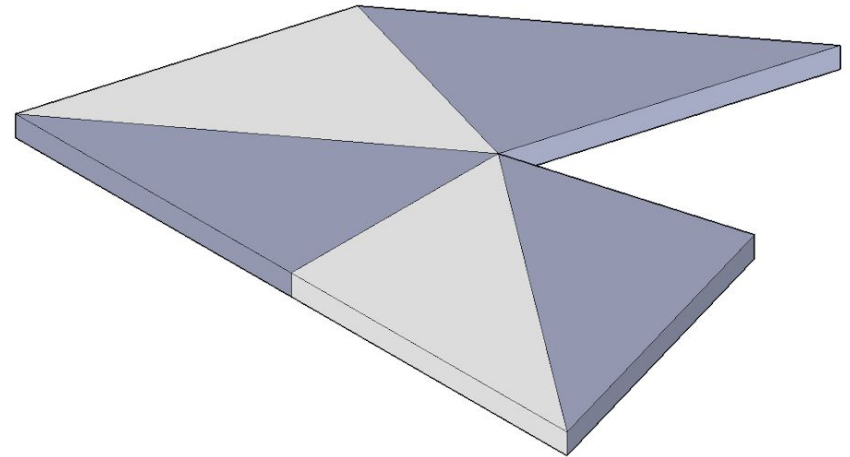
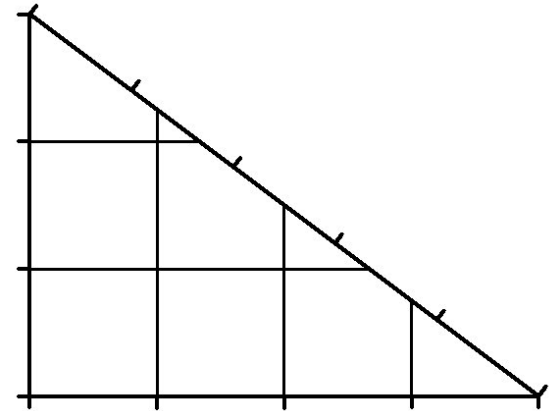
The 3-4-5 triangle is right-angled  
(because  $3^2 + 4^2 = 5^2$ )

Can you take FIVE of these triangles  
and make a symmetrical figure.

That is, one where the left half is a  
mirror image of the right half.

there are no holes in it.

The triangles must lie flat  
on the table. They may be  
rotated or turned over,  
but may not overlap.



(this is, obviously, not the right  
answer)

# The "3-4-5 Symmetry" Puzzle

Does it fit the four criteria?

YES!

Challenging?

– yes, it takes quite a while

Easy to make?

– yes, all the pieces are the same

Nicely presented?

– it can sit in a holder, like this

A surprise or "Aha!" moment?

– oh, yes

(copies available from me, either card or hardboard)

If you solve it, please keep it hidden.

