Hannah's Granny's Humbugs

A sequel to "Hannah's Sweets" saga – Summer 2015

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Recall: "Hannah's Sweets" saga Summer 2015

Mirror

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GCSE maths students vent fury over exam question about sweets they claim was 'too hard'

More than 5,000 people have signed a petition urging exam board Edexcel to lower its grading boundaries following a question about sweets

A tricky maths GCSE question about sweets has sparked a social media backlash from students claiming the exam was too hard.

The equation in yesterday's paper by exam board Edexcel involved the probability of 'Hannah' pulling two orange sweets from a bag.

Within just hours of the exam, complaints from frustrated students had gone viral on Twitter

Hannah consulted her Granny.

Granny adapted Hannah's original problem and confined her attention to its probability aspects.

She worked first with the contents of a bag of a total of n "humbugs".

Visualise first the n humbugs in wrappers labelled $h_1, h_2, h_3, \ldots, h_n$.

Ordered-pairs could be identified "with replacement" in n^2 ways.

Specifying "without replacement" removes from the reckoning the n sample positions on the leading diagonal leaving $(n^2 - n)$ admissible possibilities

		First draw				
S		h_1	h_2	h_3	...	h_n
a d	h_1	x	✔	✔	....	✔
c r	h_2	✔	x	✔	....	✔
o a	h_3	✔	✔	x	....	✔
n w	...	....	...	...	x	...
d	h_n	✔	✔	✔	....	x

The n "humbugs" in the bag comprised m described as "striped" and $(n-m)$ described as "golden" humbugs".

Granny focussed on the subset of the m "striped humbugs". Reasoned as before, sampling ordered-pairs "without replacement" yielded (m^2-m) admissible possibilities.

Thus the probability of drawing a pair of "humbugs" that were "striped" was

$$\text{prob} = \frac{m^2-m}{n^2-n}.$$

When re-expressed and factorised, this became:

$$\text{prob} = \frac{m^2-m}{n^2-n} = \frac{m\,(m-1)}{n\,(n-1)} = \frac{m}{n} \times \frac{(m-1)}{(n-1)}.$$

To Granny, this derivation seemed preferable to that dependent on the "plausible", but to the novice not always convincing, reasoning which presumes an understanding of "independence" of events and the subsequent requirement for the "multiplication of their probabilities".

Next, Granny dreamt that she was on her way to "Humbug Heaven.

The contents of k bags identical to that previously discussed were tipped into a sampling container. The total number of "humbugs" was thus kn and the total number of striped ones was km.

The probability of drawing a pair of "humbugs" that were "striped" was

$$\text{prob} = \frac{k^2 m^2 - k\,m}{k^2\,n^2 - k\,n}.$$ Re-written, $$\text{prob} = \frac{m^2 - m/k}{n^2 - n/k}.$$

As $k \to \infty$, $\text{prob} \to \frac{m^2}{n^2}$.

Again, to Granny, this derivation seemed preferable to that dependent on other "plausible", but to the novice not always convincing, lines of reasoning.

All available on-line (!):

BROWN AND WHITE HUMBUGS
[£1.50-200G]

Traditional mint flavour sweets with a chewy toffee centre. Each mint is a brown and white sweet which is individually wrapped. £1.50 for 200grams

£1.50

OLD FASHIONED GOLDEN HUMBUGS
[£1.80-200G]

Traditional old fashioned humbugs is a mint flavour gold pyramid with a chewy toffee centre. £1.80 for 200grams

£1.80