

MathsJam 2016
(and U3A and G4G CoM)

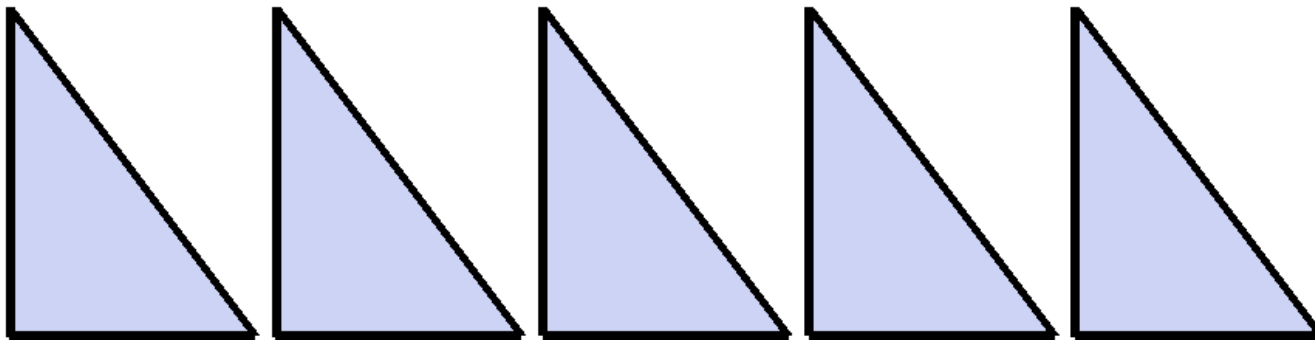
Symmetric Oddities (and Eventies)

Donald Bell
donald@marchland.org

Symmetric Oddities (and Eventies) – MathsJam 2016

Last year, I presented this puzzle:

"place these five 3-4-5 triangles side by side to make a mirror-symmetric shape. They may be rotated or turned over, but they must not overlap."



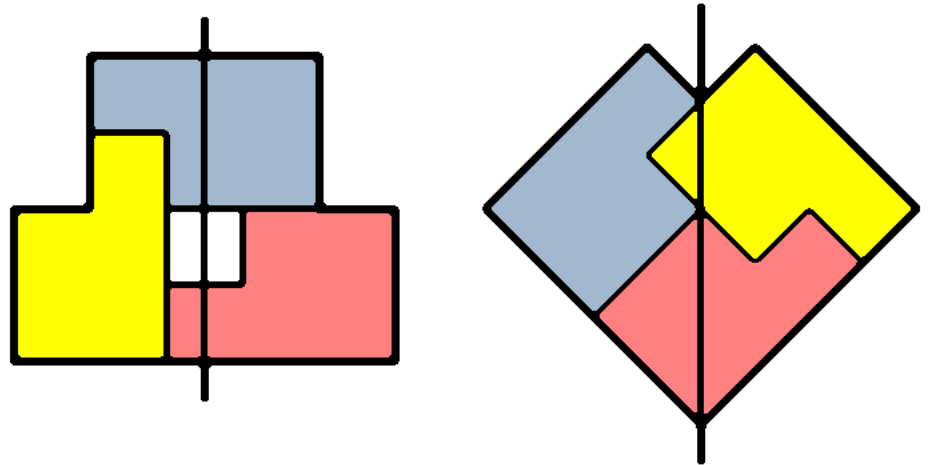
So there is an ODD number of ASYMMETRIC pieces which makes a TWO-WAY SYMMETRICAL shape (the left half is the mirror image of the right half).

The first of the "Symmetric Oddities"

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If the shape is a polyomino (ie made up of squares), then it can still make a wide variety of puzzles, some hard, some easy

Three P-pentominos can be arranged to make a symmetrical shape in several ways. Here are just two of them:

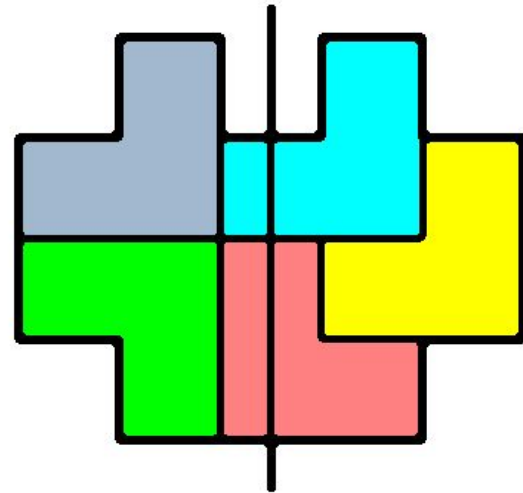


Three L-tetrominos can be arranged to make a symmetrical shape in two ways, making a nice puzzle.

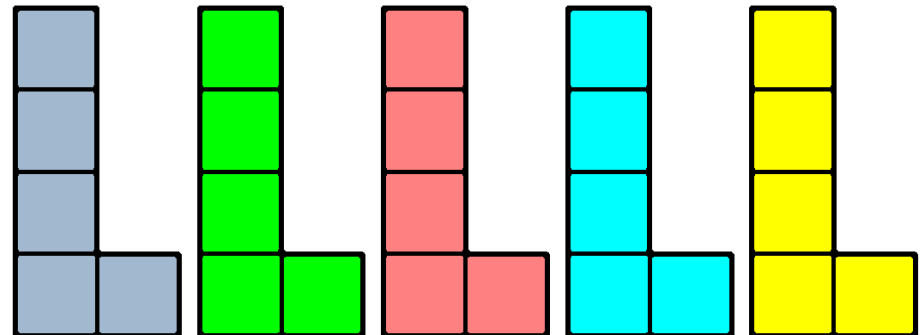
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If the (odd) number of pieces is five, then there are several more polyomino puzzles.

Five L-triominos will go together without much of a challenge



But it is very difficult to find a mirror-symmetric shape that can be made with five L-pentominos



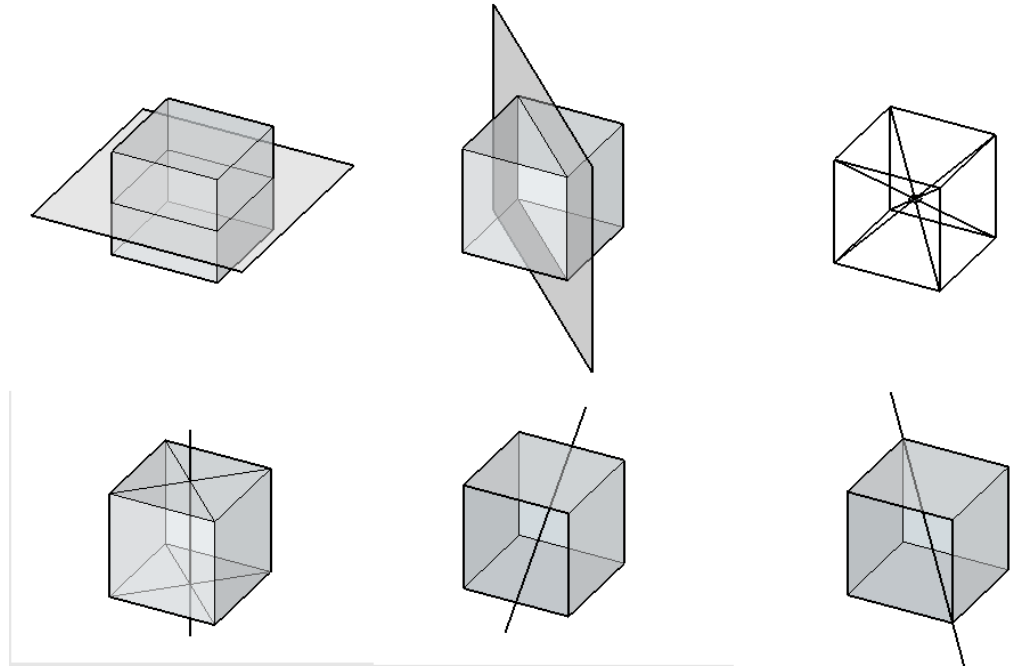
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So, in TWO dimensions, it is possible to make a TWO-way symmetrical shape using an ODD number of identical ASYMMETRIC pieces. What about THREE dimensions?

A cube can have six types of symmetry.

- Two mirror-image
- One point-symmetric
- Three rotational

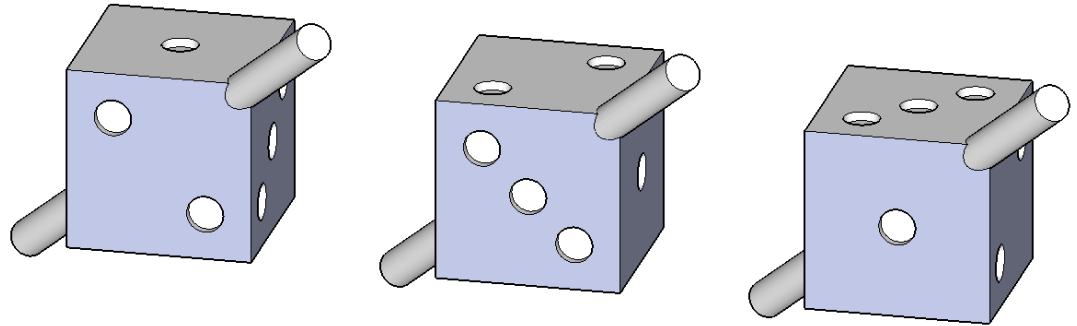
That's far too many to have to explain in a puzzle leaflet.



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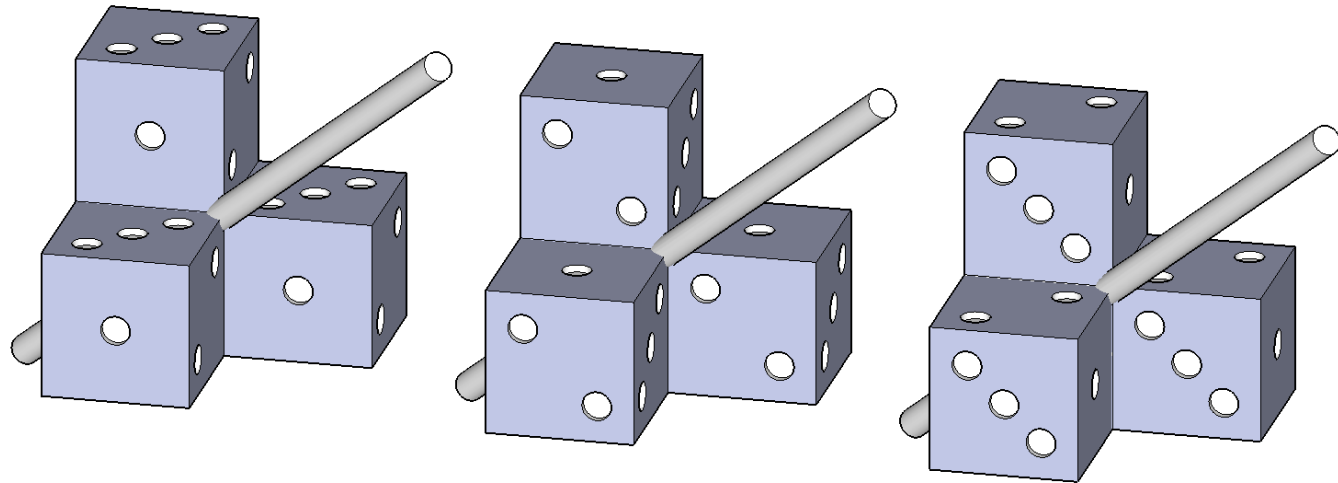
One of the rotational symmetries uses the body diagonal of the cube.

It is a **THREE**-way symmetry with 120° rotation.



A polycube can have the same kind of symmetry.

Here is one of the tetracubes (also marked up as a dice)

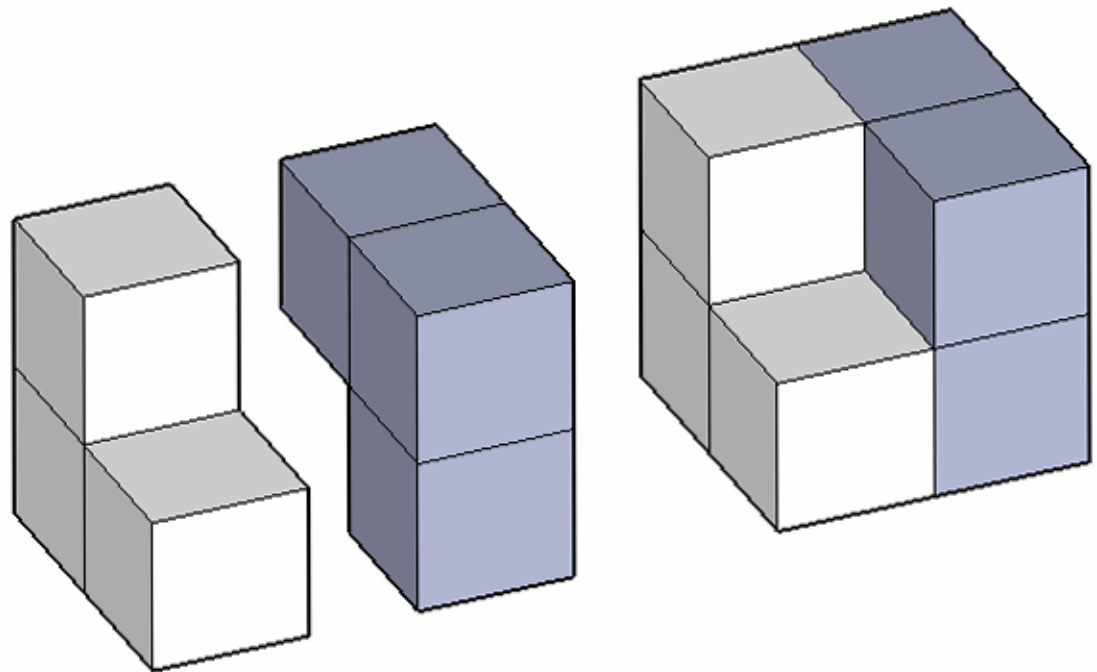


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But the "three" in three-way symmetry is an ODD number. Can there be an EVEN number of identical polycube pieces that can make a shape with ODD symmetry?

Two L-tricubes will go together to make a hexacube with three-way rotational symmetry.

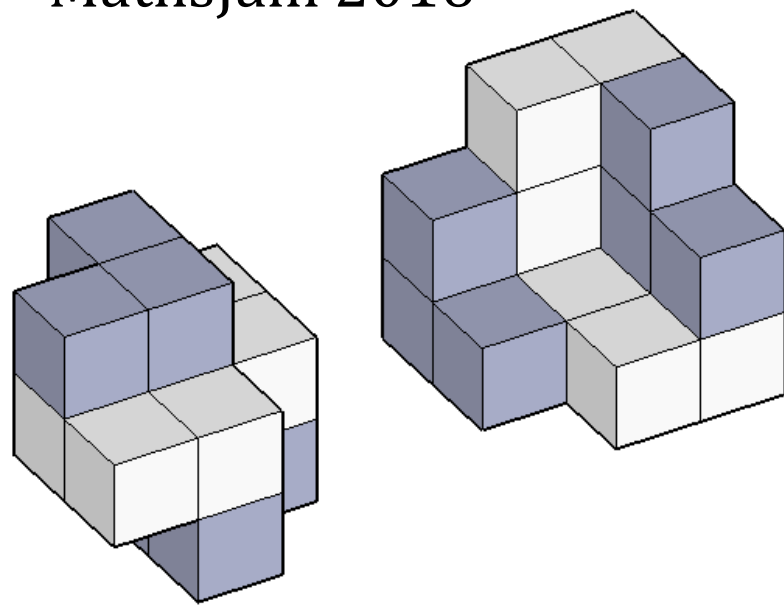
It's a $2 \times 2 \times 2$ cube with a cube missing from opposite corners.



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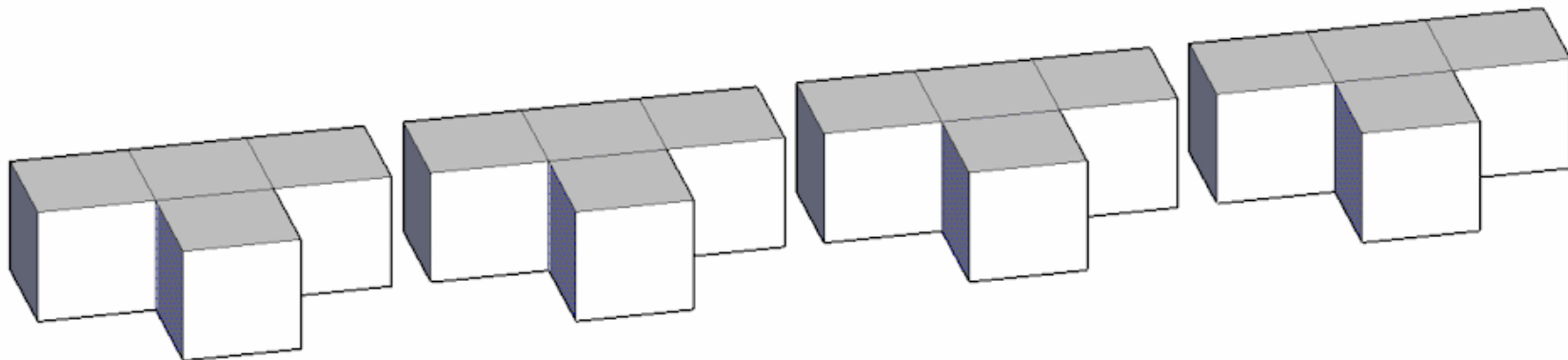
A bit more difficult:
FOUR L-tricubes making a
three-way symmetrical shape.

It is not easy to see the symmetry.



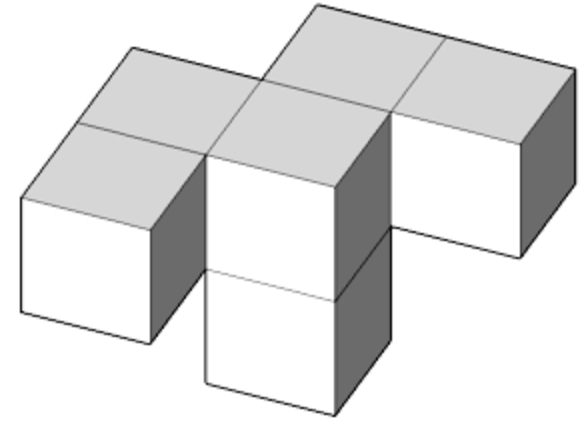
Now for something a bit harder:

Make a three-way symmetrical shape using four T-tetracubes



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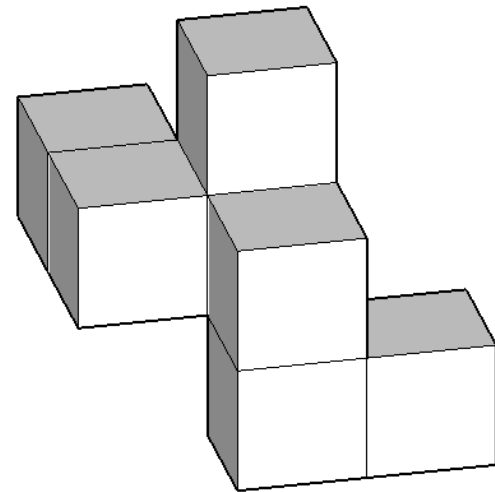
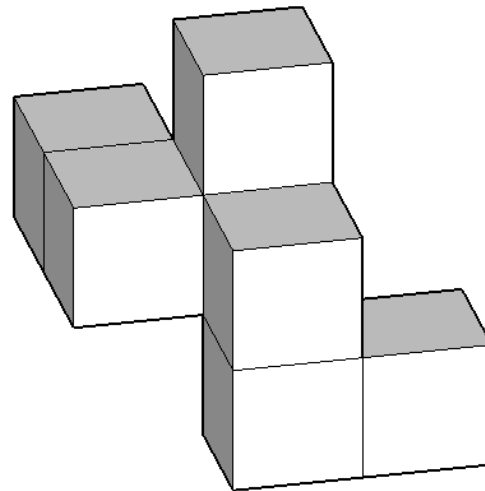
If the polycube pieces are a bit bigger, it is possible to make a decent puzzle with only two pieces.



Two of these "bumpy-W" hexacube pieces can make a three-way symmetric shape.

And this is a bit harder:

Make a three-way symmetrical shape using two heptacubes, like this:



(samples of all these puzzles are available on the tables)