Numerical frieze patterns

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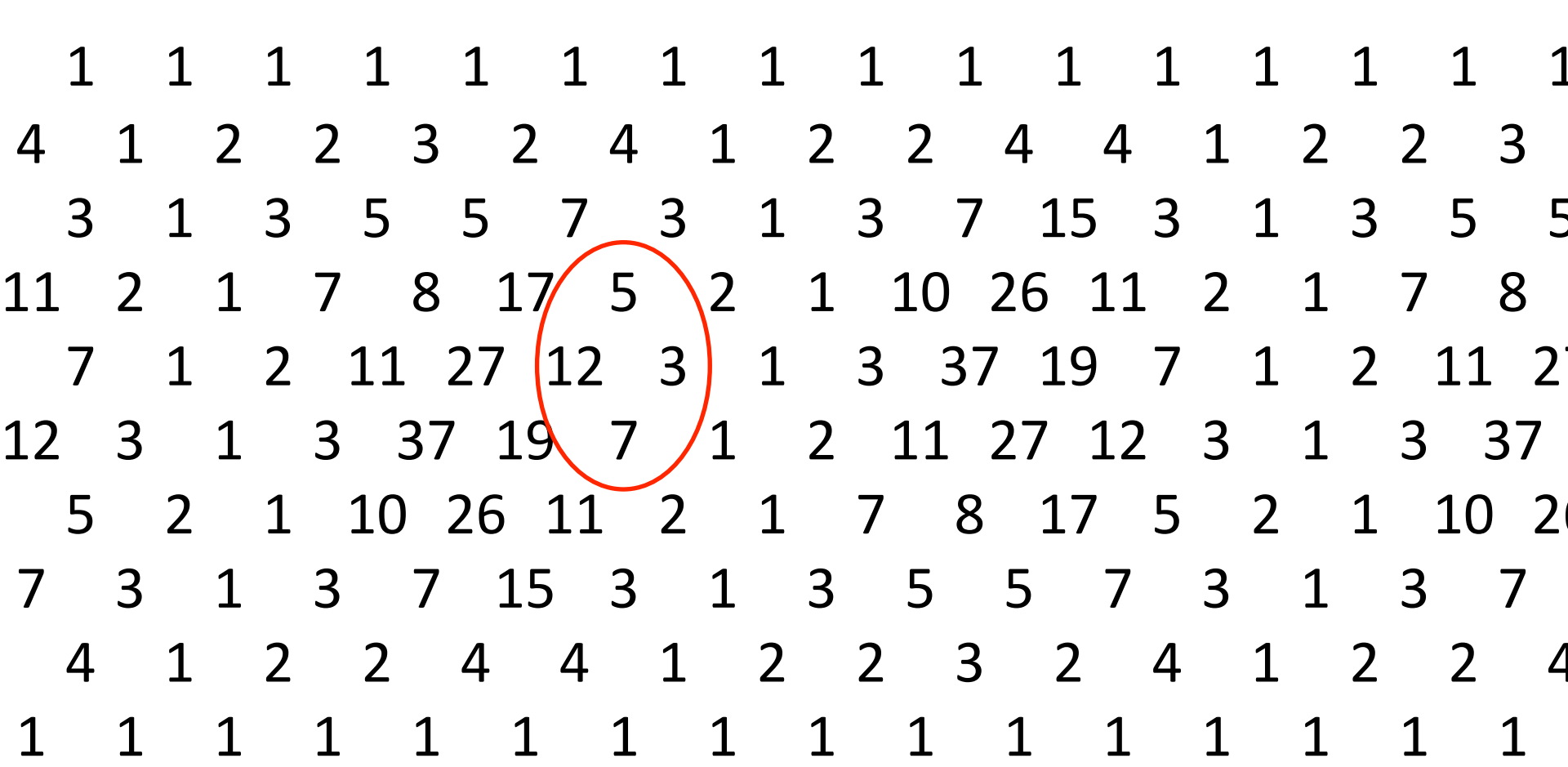
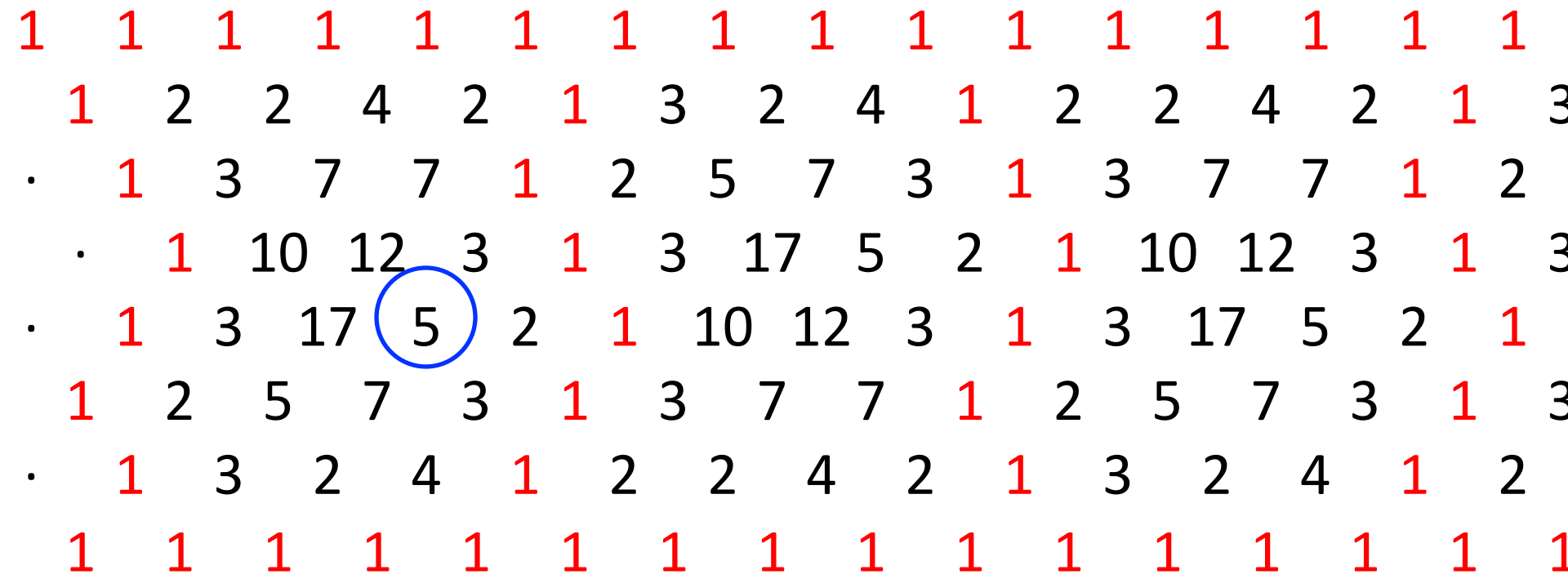


- What is the pattern?

$$\begin{array}{ccc} & a & \\ b & & c \\ & d & \end{array}$$

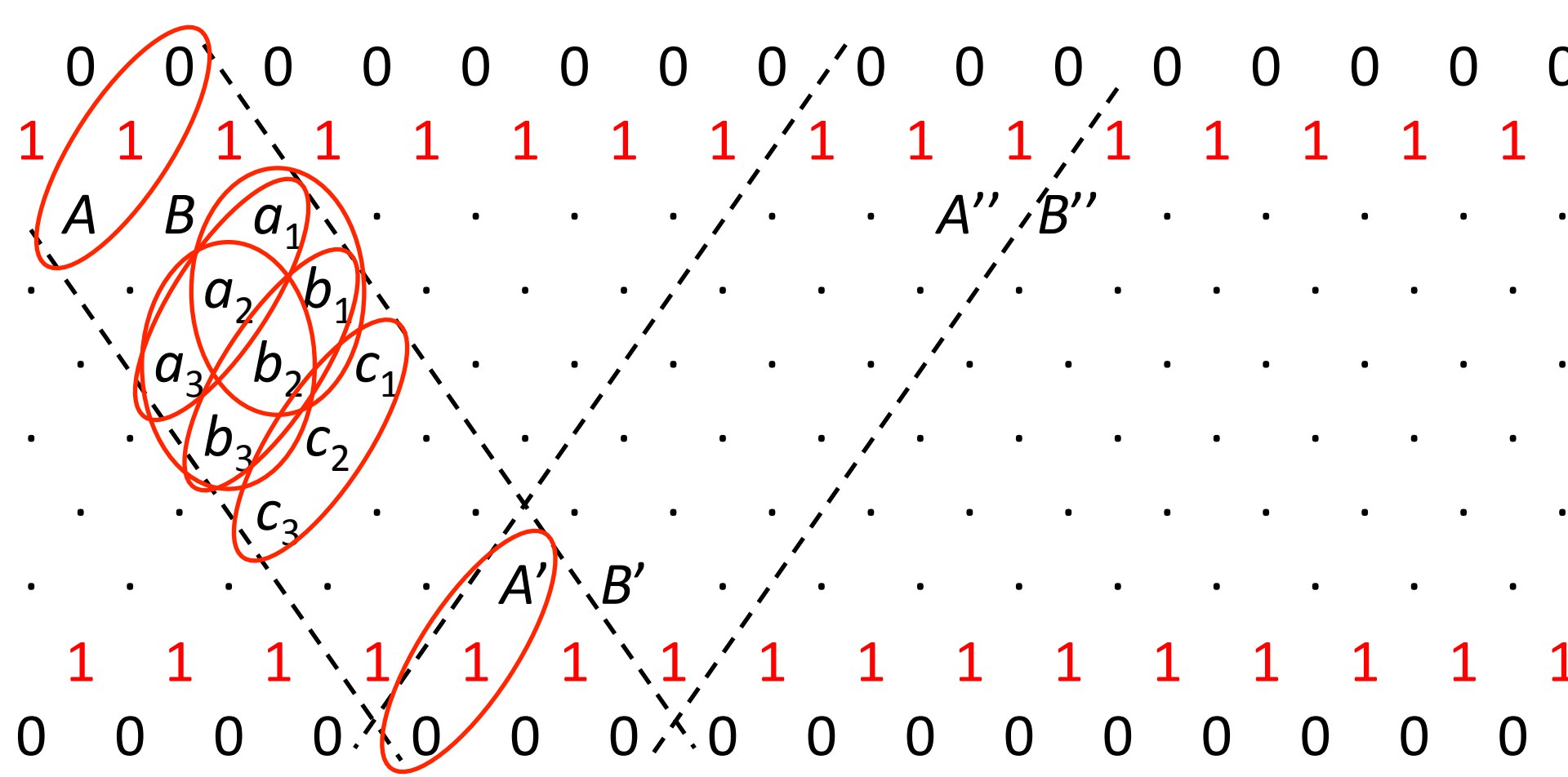
$$bc = ad + 1$$



- Frieze pattern with glide symmetry

$$\begin{array}{ccc} & a & \\ b & & c \\ & d & \end{array}$$

$$bc = ad + 1$$



$$a_2b_1=a_1b_2+1 \Rightarrow (a_1+a_3)/a_2=(b_1+b_3)/b_2$$

$$a_3b_2=a_2b_3+1 \qquad =(c_1+c_3)/c_2 \ldots$$

$$\therefore A=A'=A'' \ldots \quad B=B'=B'' \ldots$$

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

1 4 1 4 2 1 3 3 2 1 4 1 4 2 1 3

1 3 3 3 7 1 2 8 5 1 3 3 3 7 1 2

2 2 8 5 3 1 5 13 2 2 2 8 5 3 1 5

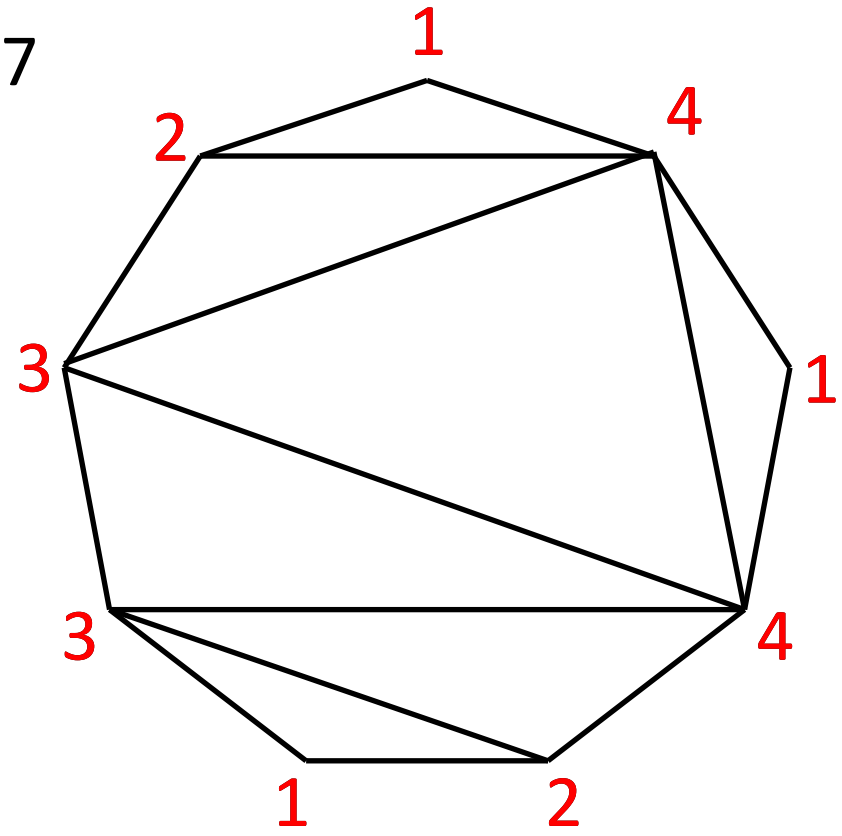
3 1 5 13 2 2 2 8 5 3 1 5 2 2 2

1 2 8 5 1 3 3 3 7

2 1 3 3 2 1 4 1 4

1 1 1 1 1 1 1 1

- Which sequences generate frieze patterns?
- Those which arise from triangulations of a polygon.



1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

1 4 1 4 2 1 3 3 2 1 4 1 4 2 1 3

1 3 3 3 7 1 2 8 5 1 3 3 3 7 1 2

2 2 8 5 3 1 5 13 2 2 2 8 5 3 1 5

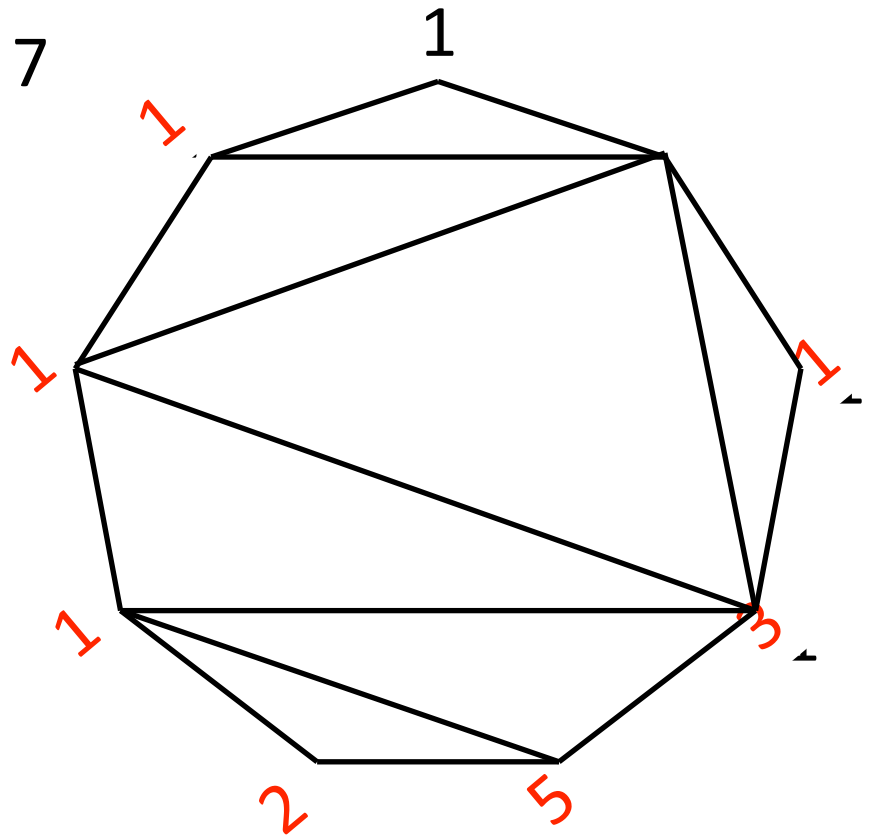
3 1 5 13 2 2 2 8 5 3 1 5 2 2 2

1 2 8 5 1 3 3 3 7

2 1 3 3 2 1 4 1 4

1 1 1 1 1 1 1 1

- Label vertex with 0
- Label neighbours with 1
- Label each vertex with sum of other 2 in triangle



Exercises

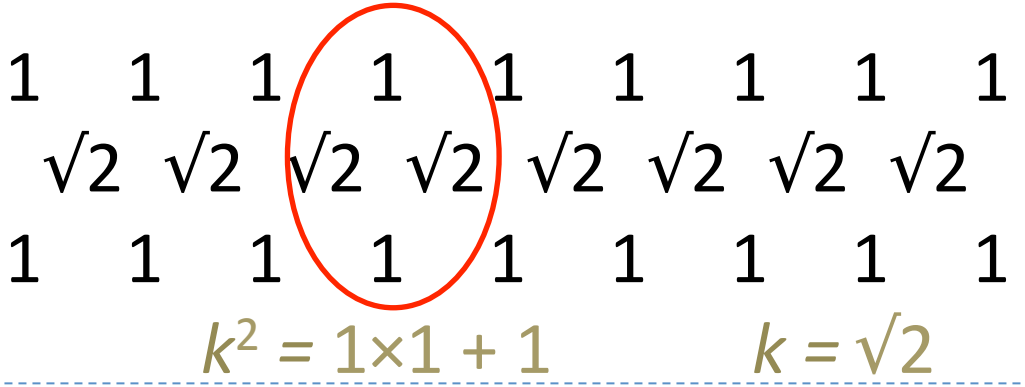
- Prove that the rule ($bc=ad+1$) yields a glide symmetry
- Prove that the pattern is generated by triangulation of a polygon
- This...

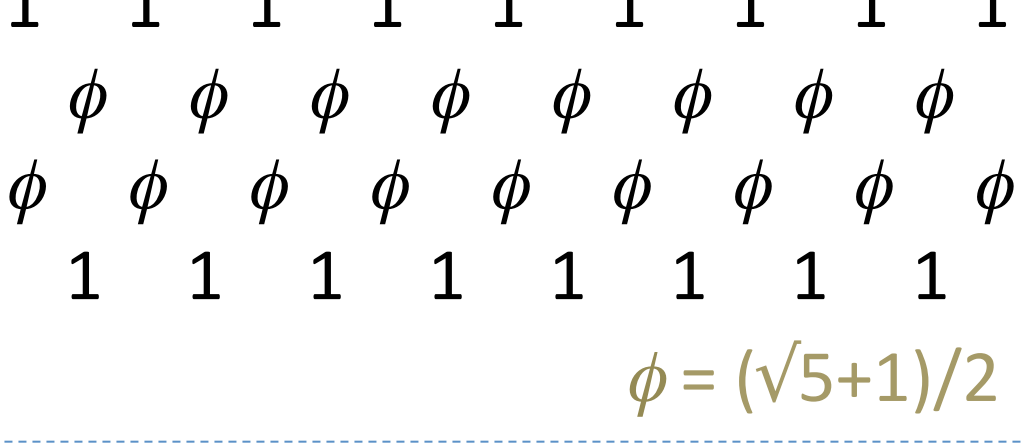
Constant values in each row

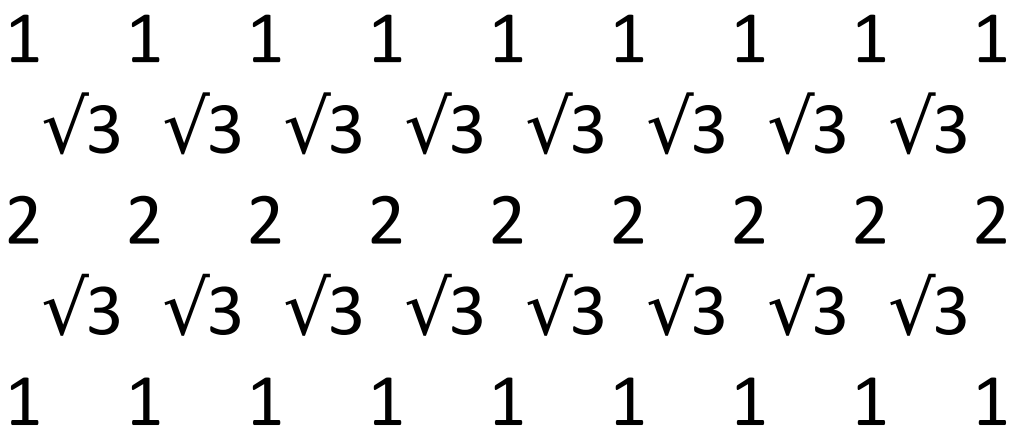
No longer integers

What is the (geometrical) significance of these values?

Can you prove it?







References

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