Ada Lovelace and Bernoulli numbers

Augusta Ada King-Noel, Countess of Lovelace (*née* **Byron**; 10 December 1815 – 27 November 1852) was an English mathematician and writer, chiefly known for her work on Charles Babbage's Analytical Engine. Her notes on the engine include what is recognised as the first algorithm intended to be carried out by a machine.

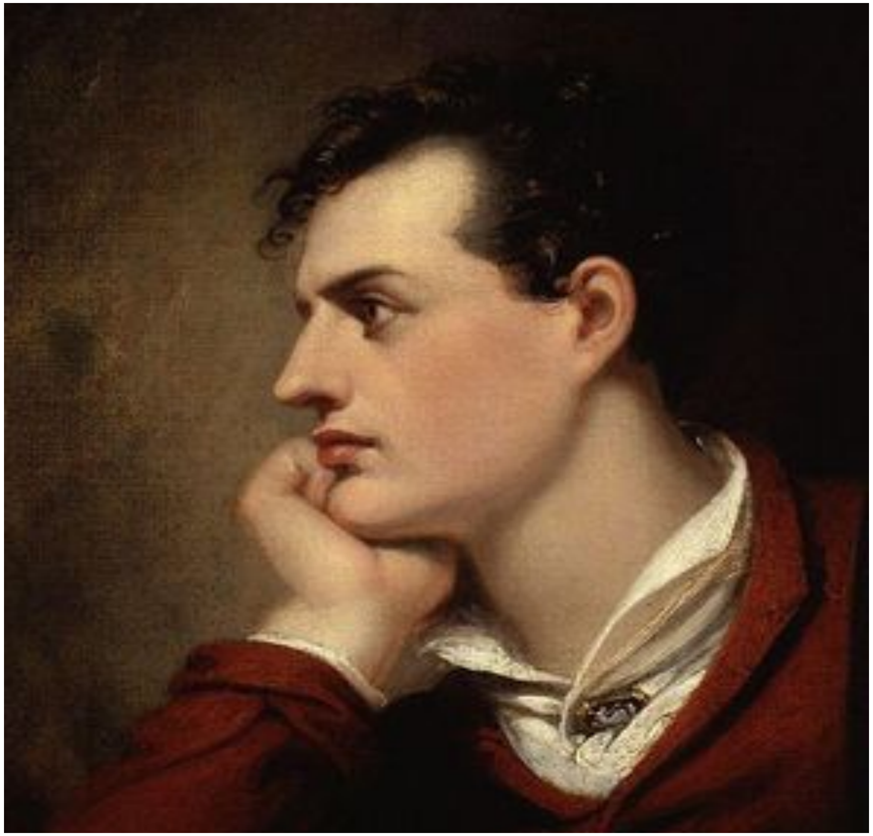
As a result, she is often regarded as the first computer programmer

@suedepom

Born Augusta Ada Byron in 1815, the only legitimate child of Lord George Gordon Noel Byron and his wife Anne Isabella (Annabella) Milbanke

Lady Caroline Lamb

Countess Teresa Guiccioli



Augusta Byron

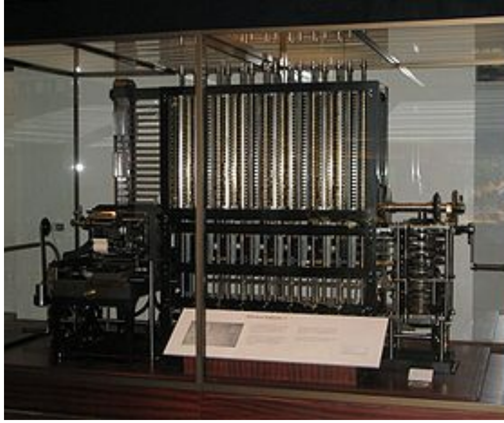
Claire Claremont

Lady Byron left her husband when Ada was eight weeks old.

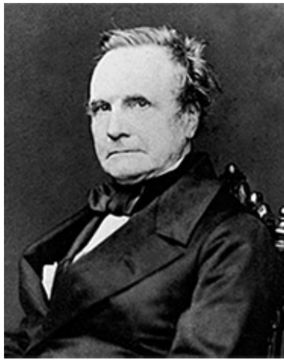
Growing up

- Isabella was determined to prevent her daughter from developing poetic tendencies and focussed her education on mathematics
- Ada had an aptitude for maths and had several tutors, including Mary Somerville
- She married, had three children in quick succession
- Within a few months of the birth of her third child in 1839, Ada decided to get more serious about mathematics again.
- Babbage suggested his friend Augustus de Morgan, professor of mathematics at University College London and noted logician.

Charles Babbage
(the Brian Cox of his day)

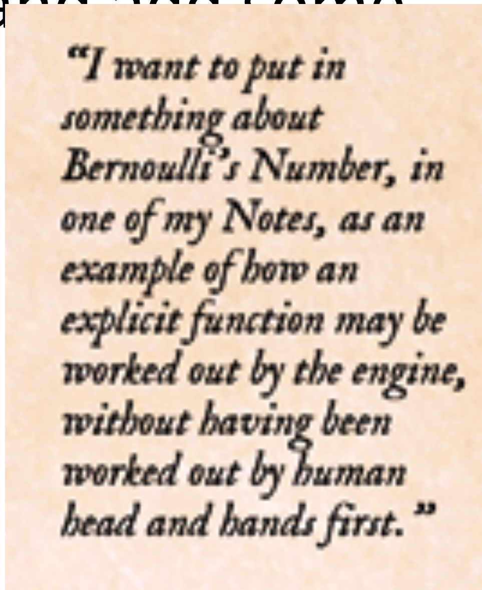


- Born Dec 26th 1791
- 1828 – 1839 Lucasian Professor of Mathematics Cambridge
- 1822 Difference Engine
- 1833 met Ada Byron when she went with her mother to see what she called his "thinking machine" a portion of his difference engine on display in his drawing room.
- 1834 Analytical Engine
- 1871 died



The first computer programme

- In 1840 Babbage spoke about his Analytical Engine at a conference in Turin in 1840
- Luigi Menabrea published a paper "*Sketch of the Analytical Engine Invented by Charles Babbage, Esq.*"
- Babbage asked Ada to translate the paper and add some notes; her notes ended up being three times as long as the original article and included the first published description of a step by step sequence of operations for solving mathematical problems



"I want to put in something about Bernoulli's Number, in one of my Notes, as an example of how an explicit function may be worked out by the engine, without having been worked out by human head and hands first."

Bernoulli numbers

$$B_0 = 1$$

$$B_1 = \pm\frac{1}{2}$$

$$B_2 = \frac{1}{6}$$

$$B_3 = 0$$

$$B_4 = -\frac{1}{30}$$

$$B_5 = 0$$

$$\sum_1^n 1 = n$$

$$\sum_1^n r = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$\sum_1^n r^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\sum_1^n r^3 = \frac{1}{3}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$

$$\sum_1^n r^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 + 0n^2 - \frac{1}{30}n$$

$$\sum_1^n r^5 = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 + 0n^3 - \frac{1}{12}n^2$$

Bernoulli numbers

$$\sum_{1}^{n} r^p = \sum_{0}^{p} \frac{B_k}{k!} \frac{p!}{(p-1+k)!} n^{p+1-k}$$

- Ramanujan wrote about them in his first paper for the Indian Mathematical Society
- The Bernoulli numbers also appear in the
  - Taylor series expansions of tan and tanh
  - Euler–Maclaurin formula
  - expressions for certain values of the Riemann zeta function.
- The generating function is the exponential function
- They can also be defined by a contour integral

Dialogue

- **A.L.L.** “I want to put in Bernoulli’s numbers to show an implicit function can be worked out by the engine without human head & hands first. Give me the necessary formulae.”
- **C. B.** “This she sent back to me ... having detected a grave mistake which I had made....”
- **A.L.L.** “Table & Diagram are seriously wrong. I have done them over in a beautiful manner, much improved.”

To solve

$$m\,x + n\,y = d$$
$$m'\,x + n'\,y = d'$$

for x and y

Columns on which are inscribed the primitive data	Number of the operations	Cards of the operations: No. of the Operation-cards	Cards of the operations: Nature of each operation	Variable cards: Columns acted on by each operation	Variable cards: Columns that receive the result of each operation	Variable cards: Indication of change of value on any column	Statement of results
$^1V_0 = m$	1	1	×	$^1V_0 \times {}^1V_4 =$	$^1V_6$	$\left\{ \begin{array}{l} ^1V_0 = {}^1V_0 \\ ^1V_4 = {}^1V_4 \end{array} \right\}$	$^1V_6 = mn'$
$^1V_1 = n$	2	"	×	$^1V_3 \times {}^1V_1 =$	$^1V_7$	$\left\{ \begin{array}{l} ^1V_3 = {}^1V_3 \\ ^1V_1 = {}^1V_1 \end{array} \right\}$	$^1V_7 = m'n$
$^1V_2 = d$	3	"	×	$^1V_2 \times {}^1V_4 =$	$^1V_8$	$\left\{ \begin{array}{l} ^1V_2 = {}^1V_2 \\ ^1V_4 = {}^0V_4 \end{array} \right\}$	$^1V_8 = dn'$
$^1V_3 = m'$	4	"	×	$^1V_5 \times {}^1V_1 =$	$^1V_9$	$\left\{ \begin{array}{l} ^1V_5 = {}^1V_5 \\ ^1V_1 = {}^0V_1 \end{array} \right\}$	$^1V_9 = d'n$
$^1V_4 = n'$	5	"	×	$^1V_0 \times {}^1V_5 =$	$^1V_{10}$	$\left\{ \begin{array}{l} ^1V_0 = {}^0V_0 \\ ^1V_5 = {}^0V_5 \end{array} \right\}$	$^1V_{10} = d'm$
$^1V_5 = d'$	6	"	×	$^1V_2 \times {}^1V_3 =$	$^1V_{11}$	$\left\{ \begin{array}{l} ^1V_2 = {}^0V_2 \\ ^1V_3 = {}^0V_3 \end{array} \right\}$	$^1V_{11} = dm'$
	7	2	−	$^1V_6 - {}^1V_7 =$	$^1V_{12}$	$\left\{ \begin{array}{l} ^1V_6 = {}^0V_6 \\ ^1V_7 = {}^0V_7 \end{array} \right\}$	$^1V_{12} = mn' - m'n$
	[illegible]			[illegible]	[illegible]	$^1V_{12} = {}^0V_{12}$	[illegible] $mn' - m'n$ y
1	2	3	4	5	6	7	8

the table presents a complete simultaneous view of all the successive changes which these columns severally pass through in order to perform the computation. AAL

Ref: http://www.fourmilab.ch/babbage/sketch.html

The first programme

- The Since programming languages had not been invented, Lovelace had to express this in terms of the way the Jacquard loom worked.

“*The Analytical engine weaves algebraic patterns just as the Jacquard loom weaves flowers and leaves*”

- In her notes on Ada points out that both the Jacquard loom and the Analytical Engine had the ability to automatically back up the card sequence and thereby repeat a series of instructions in what would now be called a "loop“

What Ada did next

- She continued to communicate with Babbage about mathematics
- Wrote a paper on using data to model the effect of light on plant growth to improve agricultural production with her husband.
- In the 1840 she was very unwell with, what they later discovered was uterine cancer.
- She was prescribed Laudnum and morphine. Due to the effects that the drugs had on her system, she also became interested in the impact of chemicals on the mind

Death and legacy

- Ada died from uterine cancer in London on November 27, 1852.
- She was buried next to her father in at the Church of St. Mary Magdalene in Hucknall
- She resurfaces and comes to prominence in the 20th Century when Alan Turing referred to her in several contexts including a radio broadcast

"*Let us reconsider Lady Lovelace's dictat 'It can do whatever we know how to order it to perform' …*"

Ada Lovelace
1815 - 1852