A Comic, a Painting, Triangles, and Squares

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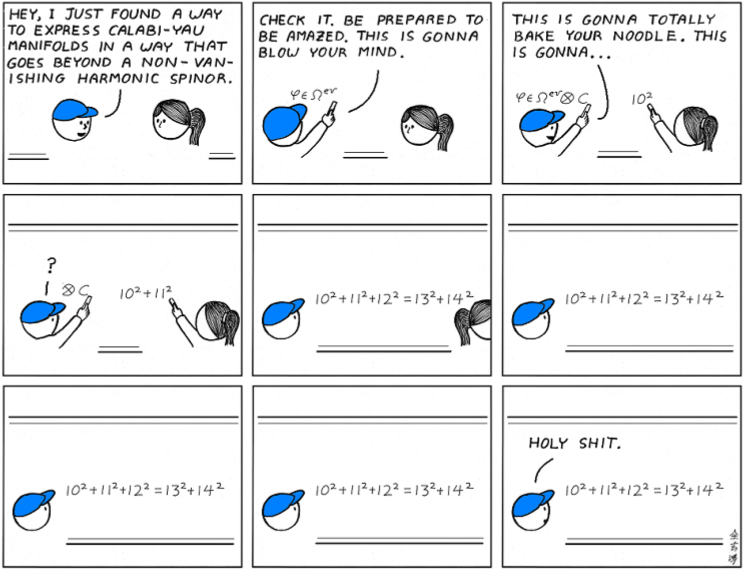
MathsJam, 18/11/2018

A comic

A comic


HEY, I JUST FOUND A WAY TO EXPRESS CALABI-YAU MANIFOLDS IN A WAY THAT GOES BEYOND A NON-VANISHING HARMONIC SPINOR.
CHECK IT. BE PREPARED TO BE AMAZED. THIS IS GONNA BLOW YOUR MIND.
THIS IS GONNA TOTALLY BAKE YOUR NOODLE. THIS IS GONNA...
?
$10^2+11^2+12^2=13^2+14^2$
HOLY SHIT.

A painting

A painting

Figure: Mental Arithmetic. In the School of M. Rachinsky by "Nikolay Bogdanov-Belsky",1896

A painting

A painting

For clarity, on the blackboard we can see:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365}$$

An equation

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$$n = 10$$

Squares: How far can we go?

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k terms on the left side, $k-1$ terms on the right side of the equality

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Observation 2:
$0 \in \mathbb{N}$

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Does it ring any bell?

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$$\triangle_{2n} = n(2n+1) = 0,3,10,21,36,55...$$

In the end, should you trust me, this will be true.

$$55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$$