The Earth-Moon Distance Question

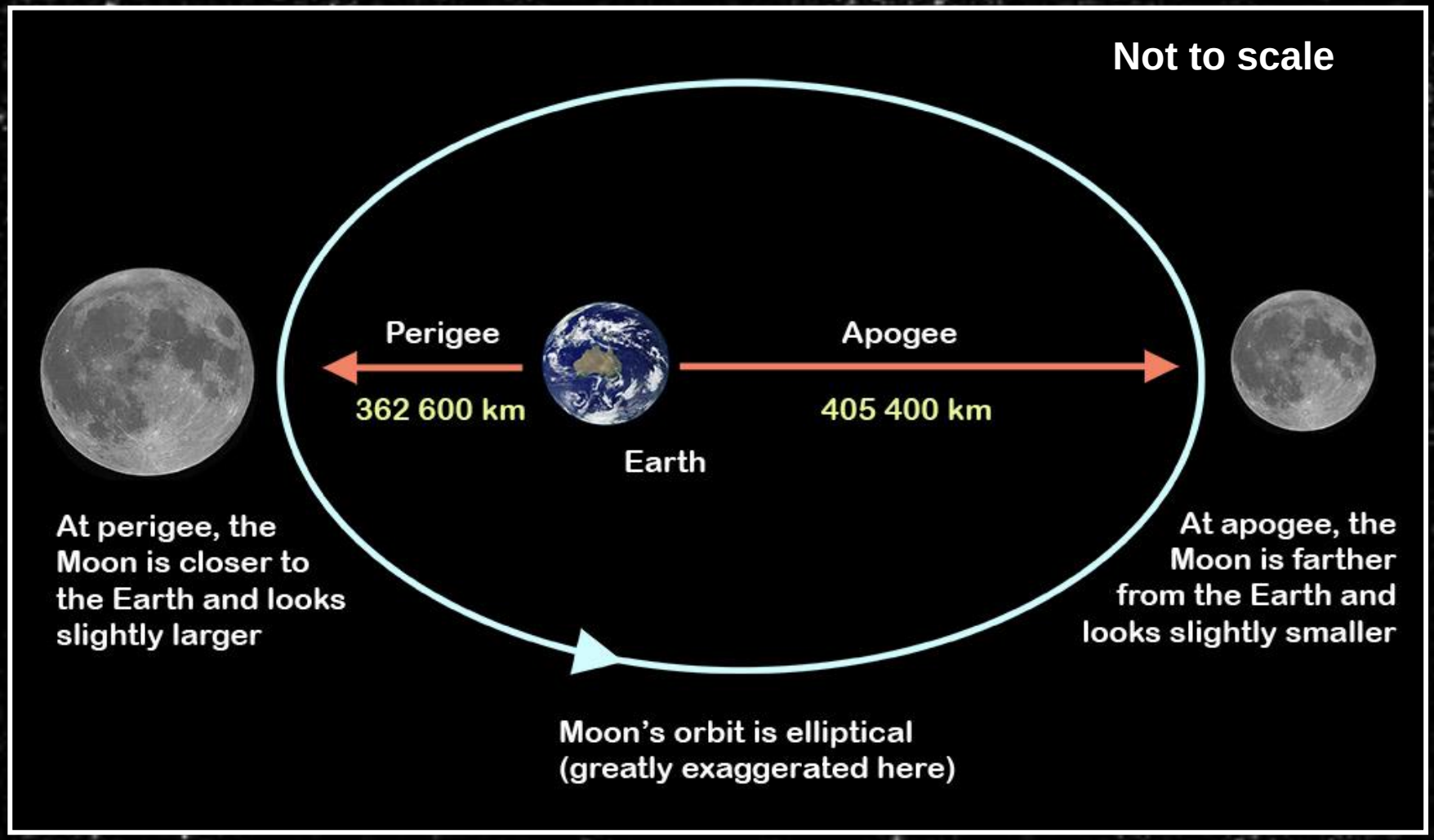
- Could you potentially fit the planets Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune into the distance between the Earth and the Moon?

- Is the answer “yes”, “no”, or “sometimes”?

Here's a clue

- This animation shows how the moon would appear over the course of one orbit of the Earth, if you could observe it continuously

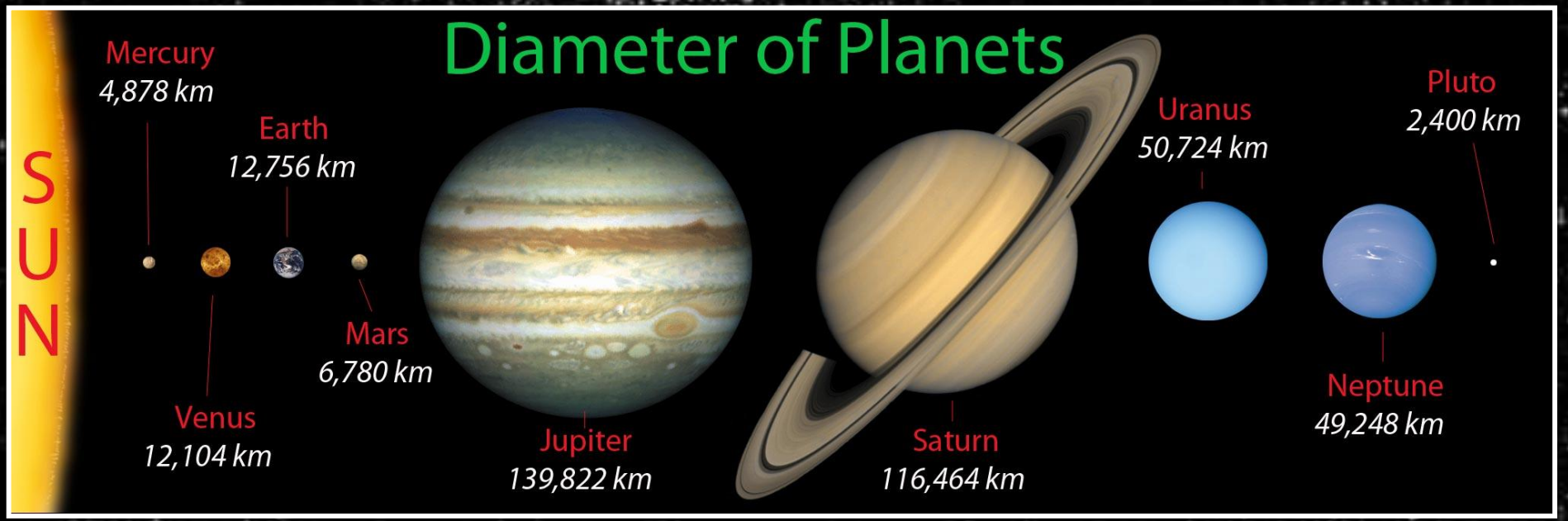
The Answer



Sometimes!

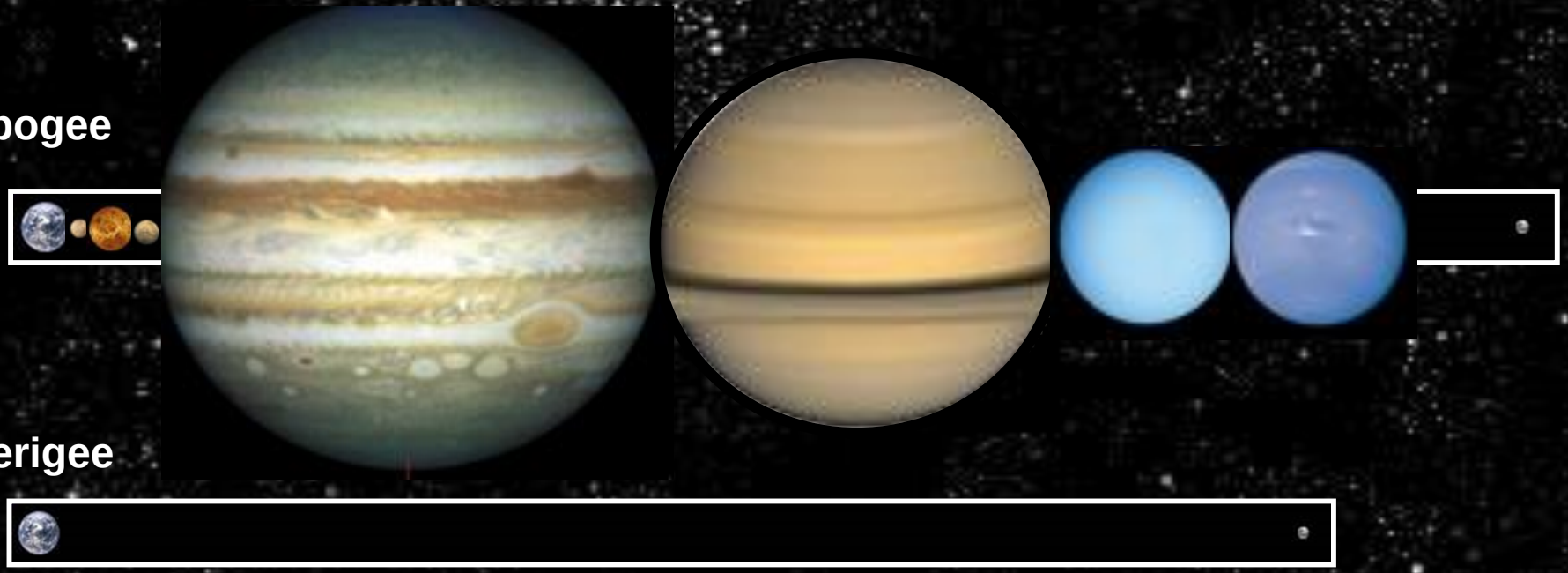
The Planets To Scale

- Adding the equatorial diameters of the planets, (except Earth), gives a total of 380,020 km



Sometimes..

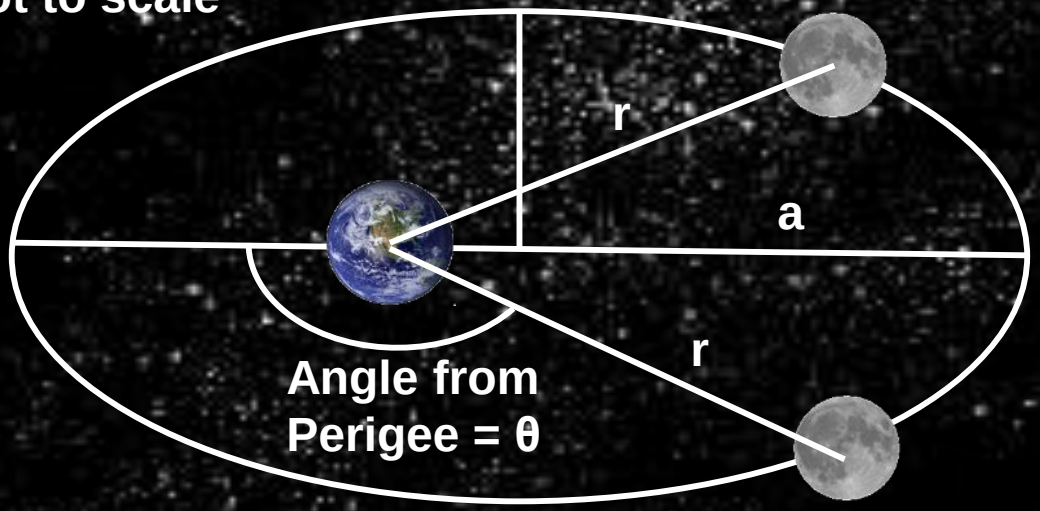
- Allowing for the radii of the Earth and Moon, the gap at apogee is 397885 km, and the gap at perigee is 348885
- So the planets fit between the Earth and the Moon when the latter is at apogee, but not when it is at perigee



How Long Is “Sometimes”?

- Calculating the angular fraction of the orbit that the planets fit is relatively easy
  - Sum of planetary equatorial diameters = 387, 941 km
  - Critical distance, (including Earth and Moon radii, 8115 km) = 396, 056 km

Not to scale



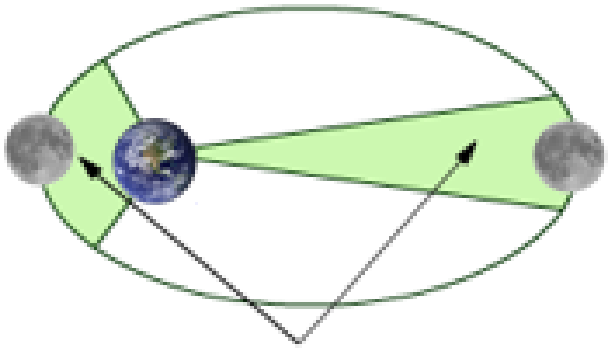
a=384,400 km
e=0.055
r= 396,056 km

Angle from Perigee = $\theta = \cos^{-1}((a(1-e^2)/er)-(1/e)) = 126^\circ$

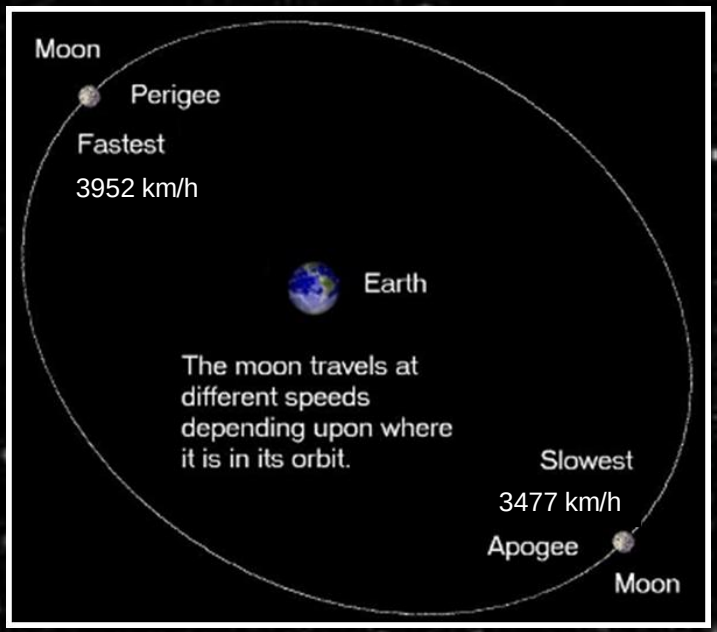
In angular terms, the planets fit for about 0.3 of the Moon’s orbit

Kepler's Second Law

- Calculating the time is somewhat trickier
- As specified by Kepler's Second Law, an imaginary line from the Earth to the Moon sweeps out equal areas in equal intervals of time
- So the velocity of the Moon around the Earth is not constant – the Moon travels faster at perigee than at apogee



The Moon sweeps out equal areas in equal intervals of time



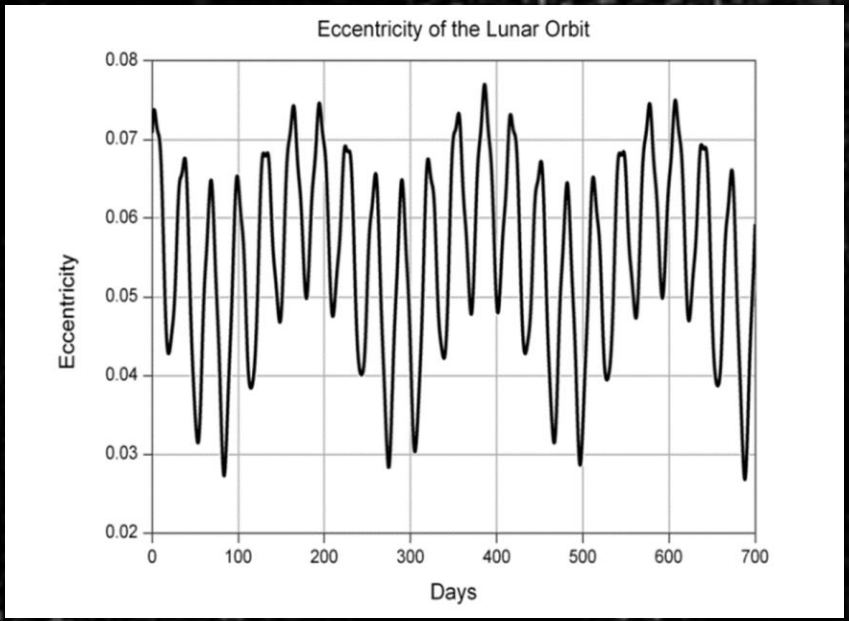
Calculating “Sometimes”

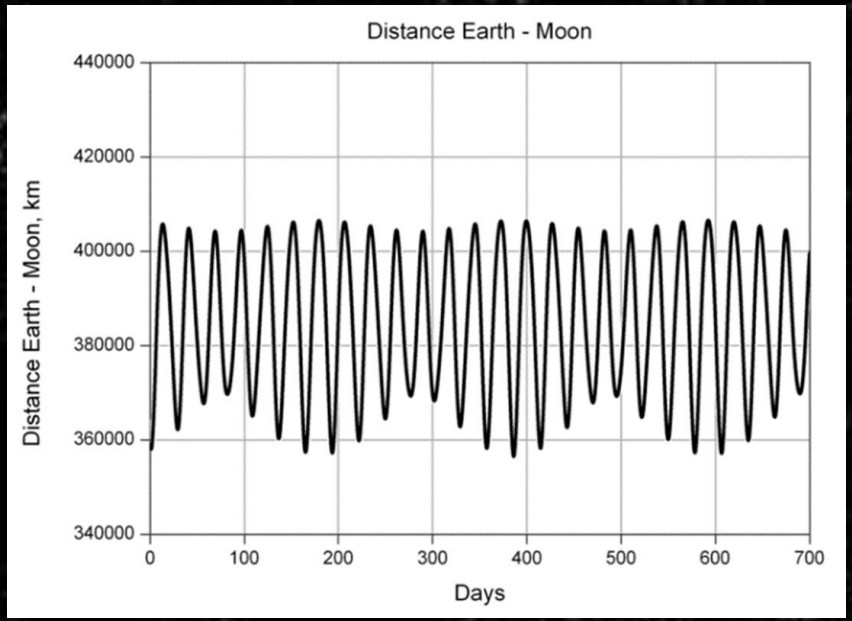
- Working in radians…..we need to know
  - The Mean Motion = n = $631.348(a^{-3/2})$ = 2.65×10^{-6} rad/s
  - The Eccentric Anomaly = E = cos-1((e+cos(θ))/(1+ecos(θ))) = 2.155 radians
  - The Mean Anomaly = M = E – esin(E) = 2.109 radians
  - The time taken to get from perigee to critical point = M/n = 796073 seconds = 9.214 days
  - The period of the Moon’s orbit = 27.32 days
- So the percentage of time for which the planets fit is:-

Fit Percentage = (27.32 –(2x9.214)/27.32)*100 = 32.55%

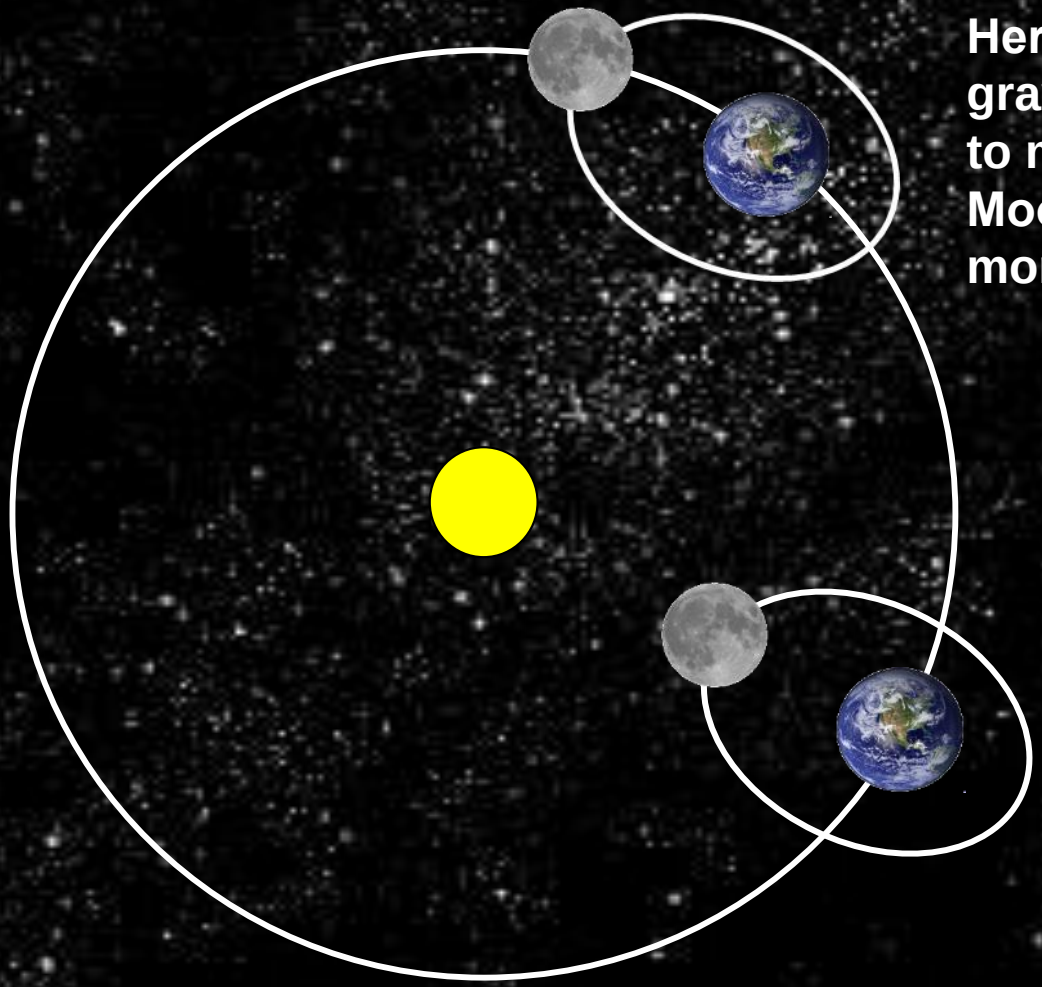
Subtleties

- The planets are somewhat oblate, (especially so in the case of Saturn), so if you turn them through 90˚, their polar diameters only sum to 364,800 km (about 6% less)
- The mean eccentricity of the Moon's orbit is 0.055, but surprisingly, it isn't constant. The Earth-Moon distance varies in accordance with this change in eccentricity





The Sun is to blame!



Here the Sun's gravity tends to make the Moon's orbit more circular

Here the Sun's gravity tends to make the Moon's orbit more elliptical

Not to scale

Conclusion

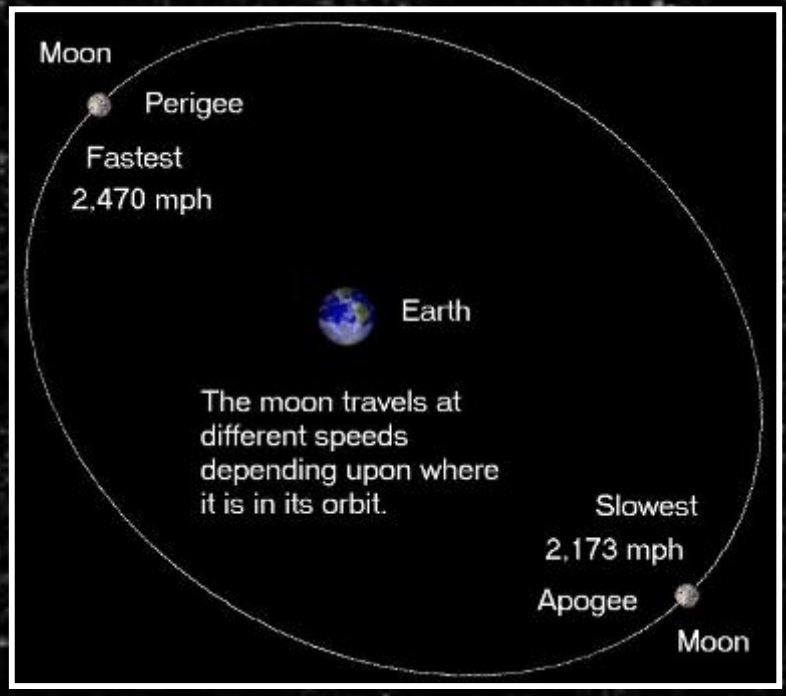
The other planets will fit between the Earth and the Moon for 32.55% of the time….sometimes!

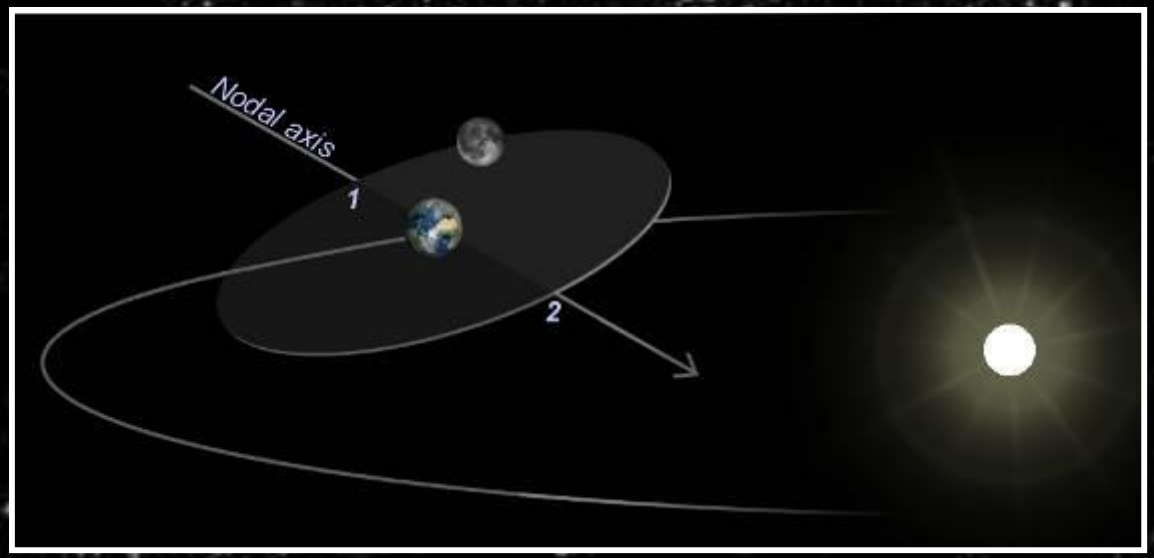
https://www.mathsisfun.com/geometry/ellipse.html

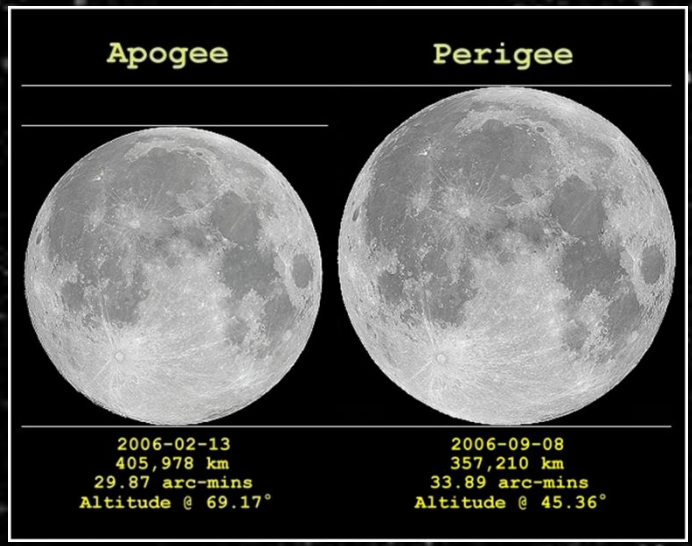
Back-Up Slides

The Moon's Orbit

- Semi-major axis~384 748 km
- Distance at perigee~364 397 km
- Distance at apogee~406 731 km
- Mean eccentricity = 0.0549006
- Mean inclination of orbit to ecliptic = 5.14°







Causes of lunar libration

Libration is a slow rocking back and forth of the Moon as viewed from Earth, permitting an observer to see slightly different regions of the surface at different times

The following are the three types of lunar libration:

Libration in longitude results from the eccentricity of the Moon's orbit around Earth; the Moon's rotation sometimes leads and sometimes lags its orbital position

Libration in latitude results from a slight inclination (about 6.7°) between the Moon's axis of rotation and the normal to the plane of its orbit around Earth. Its origin is analogous to how the seasons arise from Earth's revolution about the Sun

Diurnal libration is a small daily oscillation due to Earth's rotation, which carries an observer first to one side and then to the other side of the straight line joining Earth's and the Moon's centres, allowing the observer to look first around one side of the Moon and then around the other – since the observer is on Earth's surface, not at its centre