# Banach-Tarski: make peace with it

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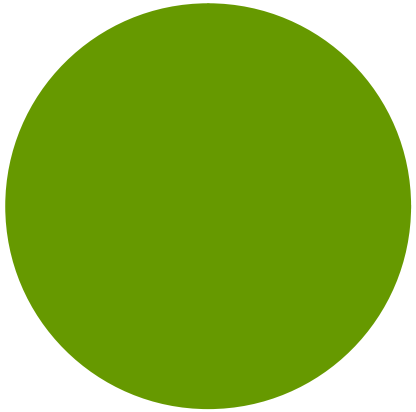
# The theorem

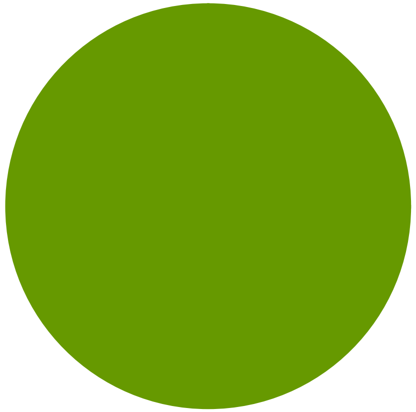
*There exists a way to divide a sphere into a finite number of pieces and rearrange those pieces into two spheres, both of which are the same size as the original.*

# Preserving the Lebesgue measure



Measure: 1

# Preserving the Lebesgue measure



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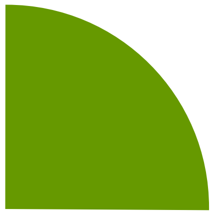
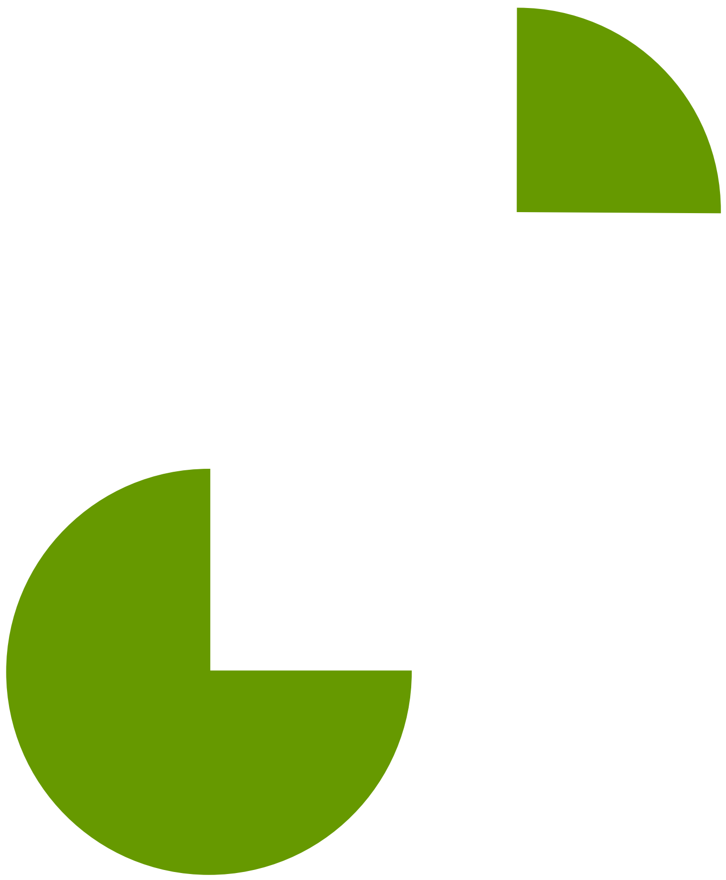


Measure: 0.25

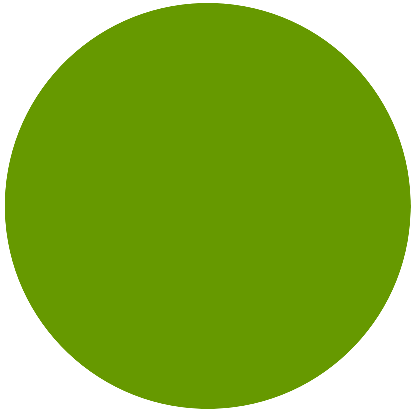
Measure: 0.75

# Preserving the Lebesgue measure



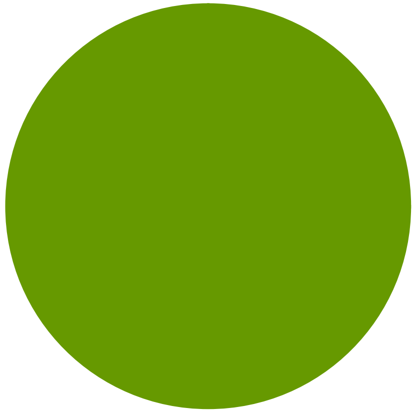
# Preserving the Lebesgue measure

$$
\begin{aligned}
&1 \\
&= 0.75 + 0.25 \\
&= 1
\end{aligned}
$$



Measure: 1

# Preserving the Lebesgue measure: loophole



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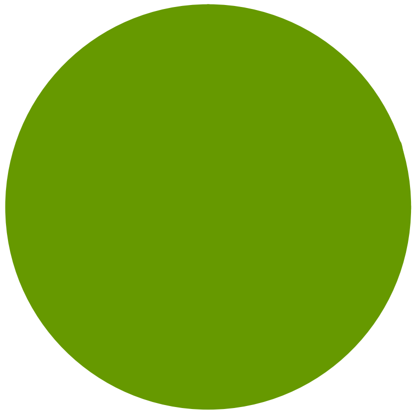

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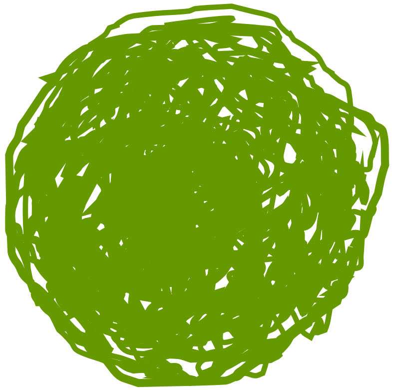

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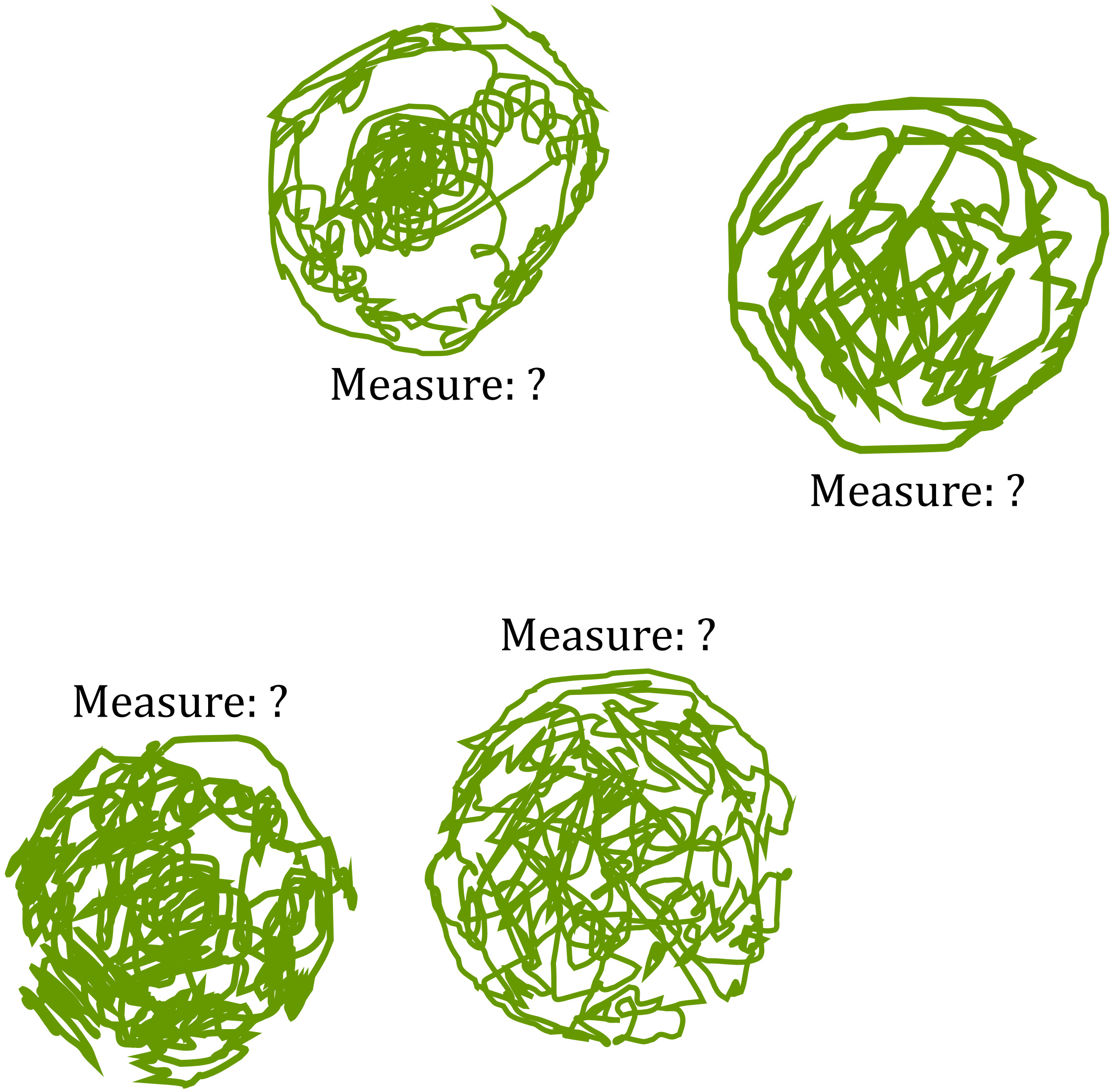

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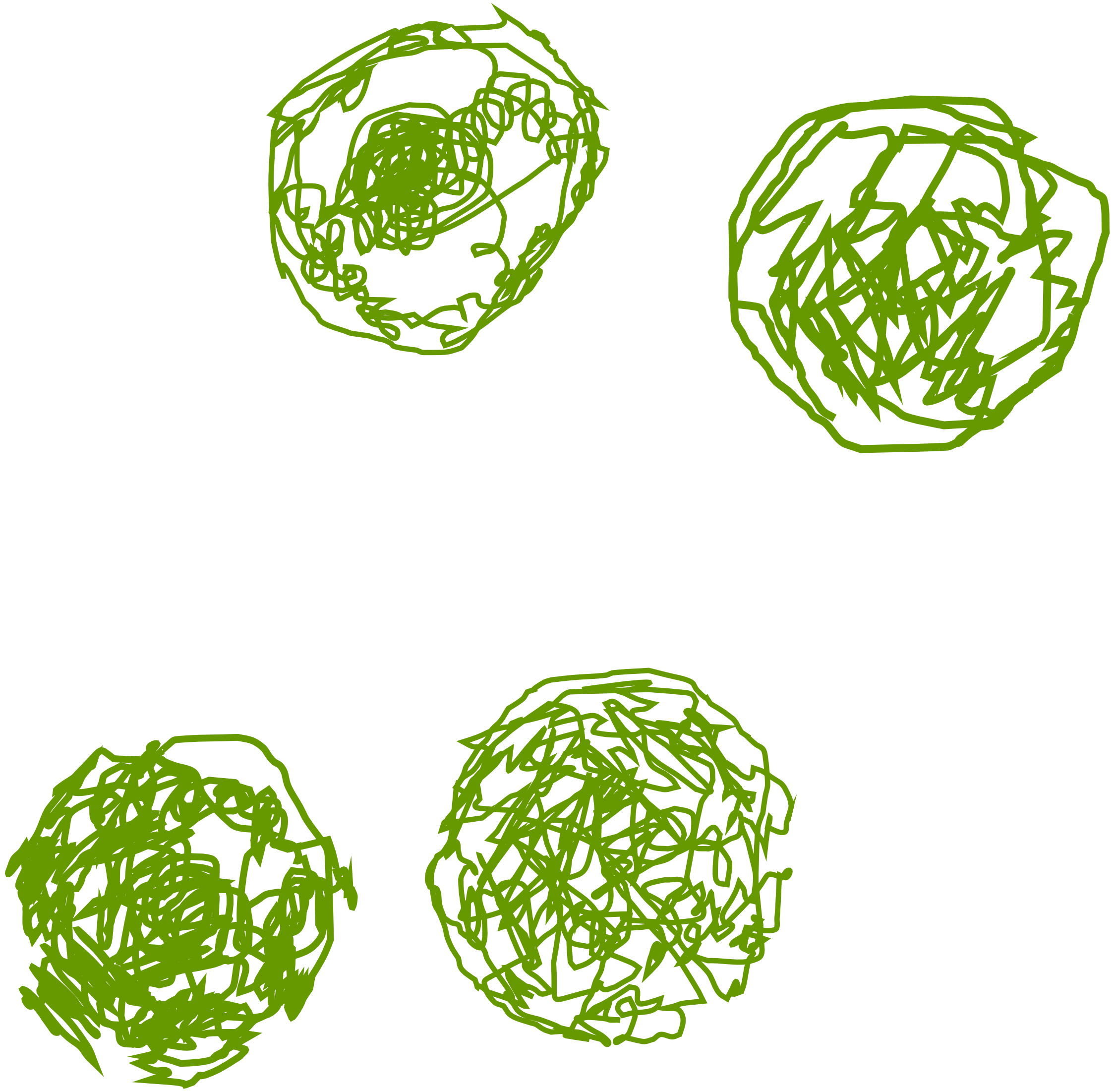

# Preserving the Lebesgue measure: loophole

$$
\begin{aligned}
&1 \\
&= ? + ? + ? + ? \\
&= 1 + 1 \\
&= 2
\end{aligned}
$$

# Why I'm not rich

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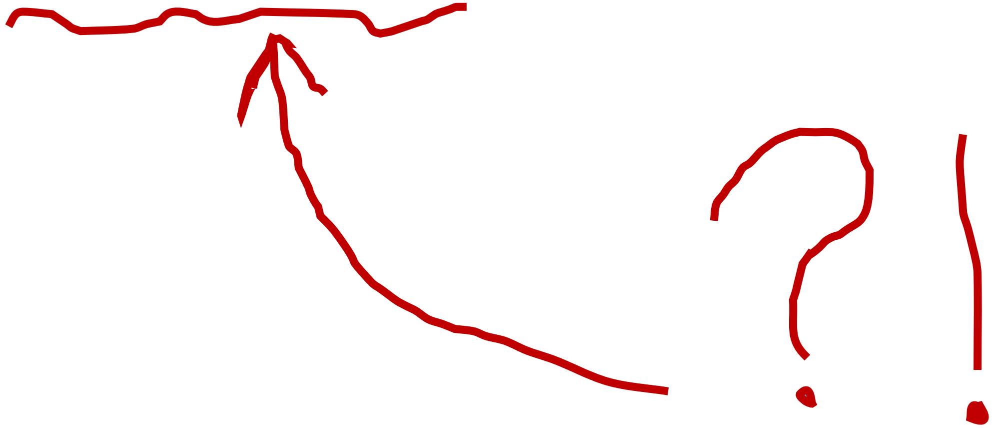

# The axiom of choice (AC)

*Given any collection of sets, each containing at least one object, it is possible to make a selection of exactly one object from each set, even if the collection is infinite.*

- Axioms of mathematics are the building blocks of sets.
- AC stands out because it doesn’t say which members are in the resultant set.

# Why can’t we know what the pieces are?

- Any proof of the paradox *must* use the axiom of choice.
- So if you knew what the pieces were, you’d be able to show the paradox without AC!

# Concluding one-liners

- The original paradox can be done in as few as five pieces.
- The pieces aren't necessarily connected.
- The paradox can be done in $n$ dimensions, $n \geq 2$.
- It can be extended to solid balls and even arbitrary shapes.
- For solid shapes, the pieces can still move smoothly without collision.