Neosemimagic Tilings

A generalisation which includes both semimagic squares and perfect squared squares brings new challenges!

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You've heard of these:

SEMIMAGIC SQUARE

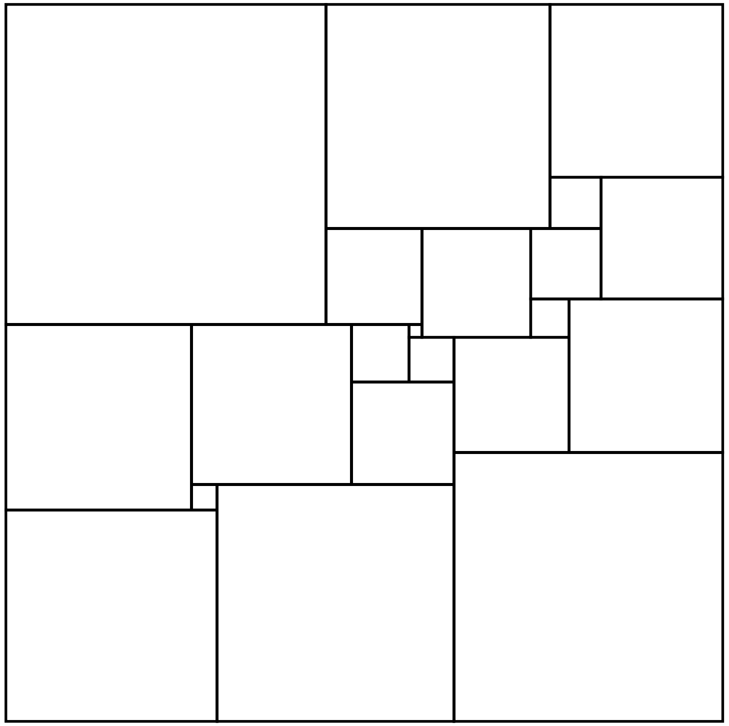
9	5	1
4	3	8
2	7	6

Distinct numbers.

Every row sum and column sum is the same (the magic sum or magic constant).

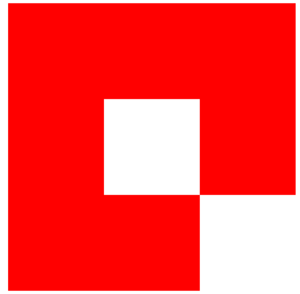
In a "normal" semimagic square the numbers are consecutive integers starting with 1.

PERFECT SQUARED SQUARE (PSS)



No two squares are the same size. The Order N is the number of squares.

POLYOMINO



A polygon, which may have polygonal holes, formed by equal-sized squares joined edge-to-edge.

But here’s something completely new:

NEOSEMIMAGIC TILING

A tiling of a polyomino by polyominoes, where

- Each tile is either empty or contains a distinct positive integer.
- The order N of the tiling is the number of non-empty tiles.
- **The sum S of the numbers in the tiles intersected by any horizontal or vertical line is the same (the magic sum or magic constant).**

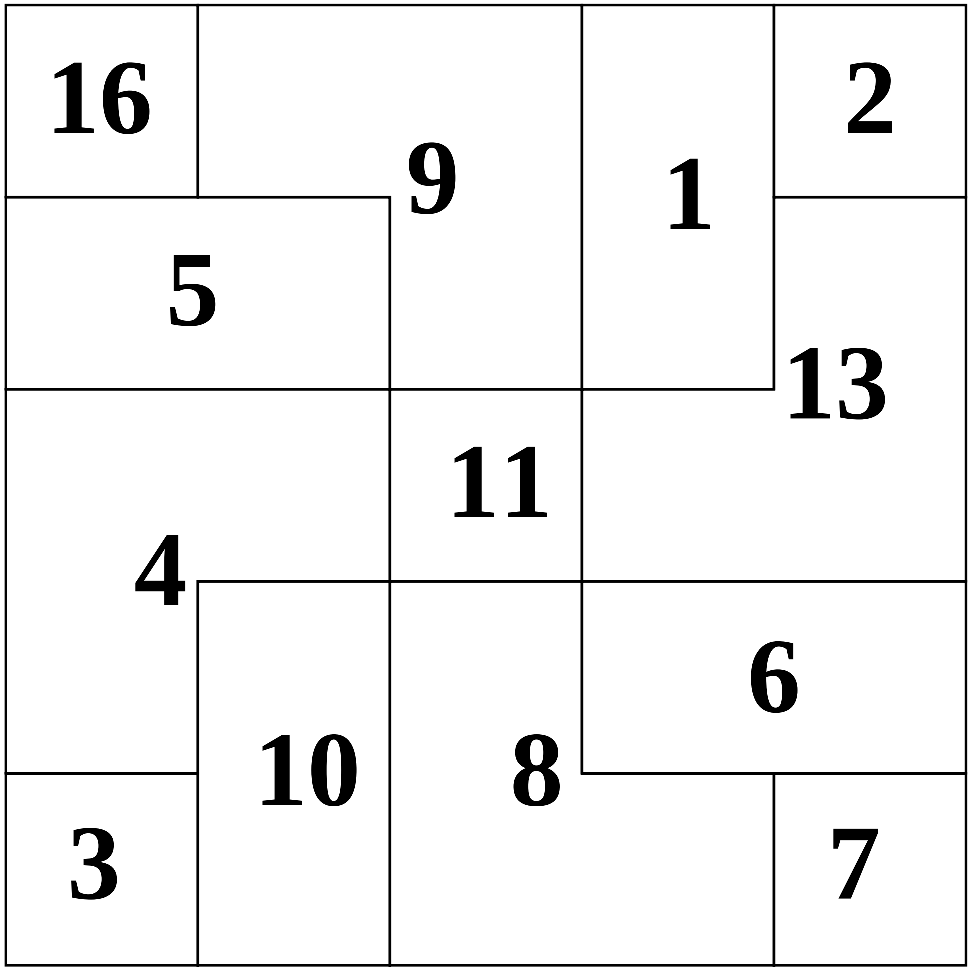
There may be fewer constraints: for example negative numbers may be allowed, but the numbers must still be distinct.

Or there may be additional constraints: for example every number must be of the same type, such as a square, cube, semiprime, or triangular number.

Every semimagic square and PSS is a Neosemimagic Square (NSMS).

EXAMPLES OF NEOSEMIMAGIC RECTANGLES (NSMRs)

N=13, S=28



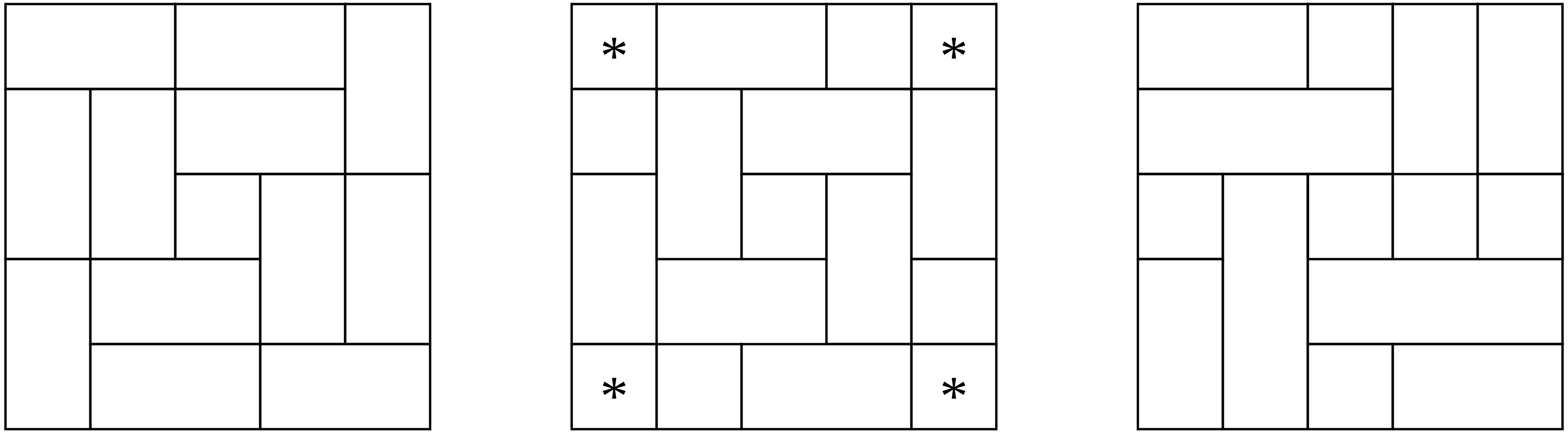
N=10, S=55250

<table>
<tr><td>176^2</td><td>153^2</td><td>24^2</td><td>17^2</td></tr>
<tr><td colspan="2">145^2</td><td colspan="2">185^2</td></tr>
<tr><td>57^2</td><td>104^2</td><td>143^2</td><td>144^2</td></tr>
</table>

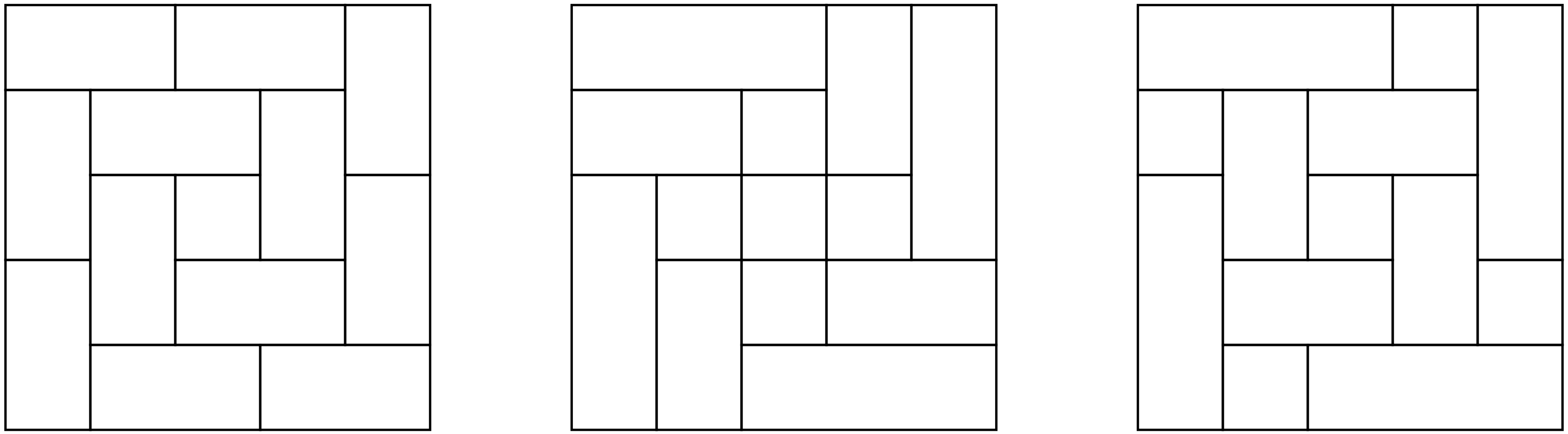
Every number is a distinct square.

CHALLENGE 1

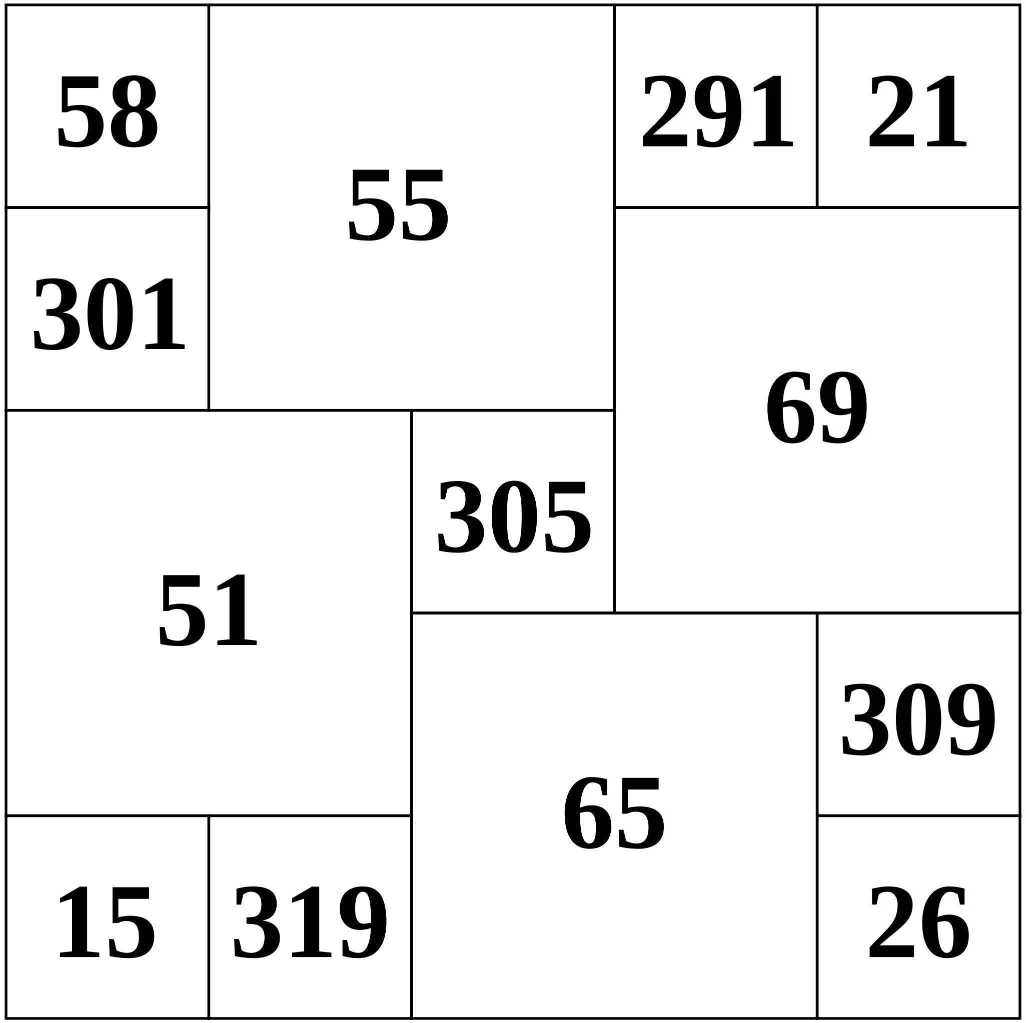
Do any of these tilings (with N=13) admit NSMSs of squares?



*: Empty tile.

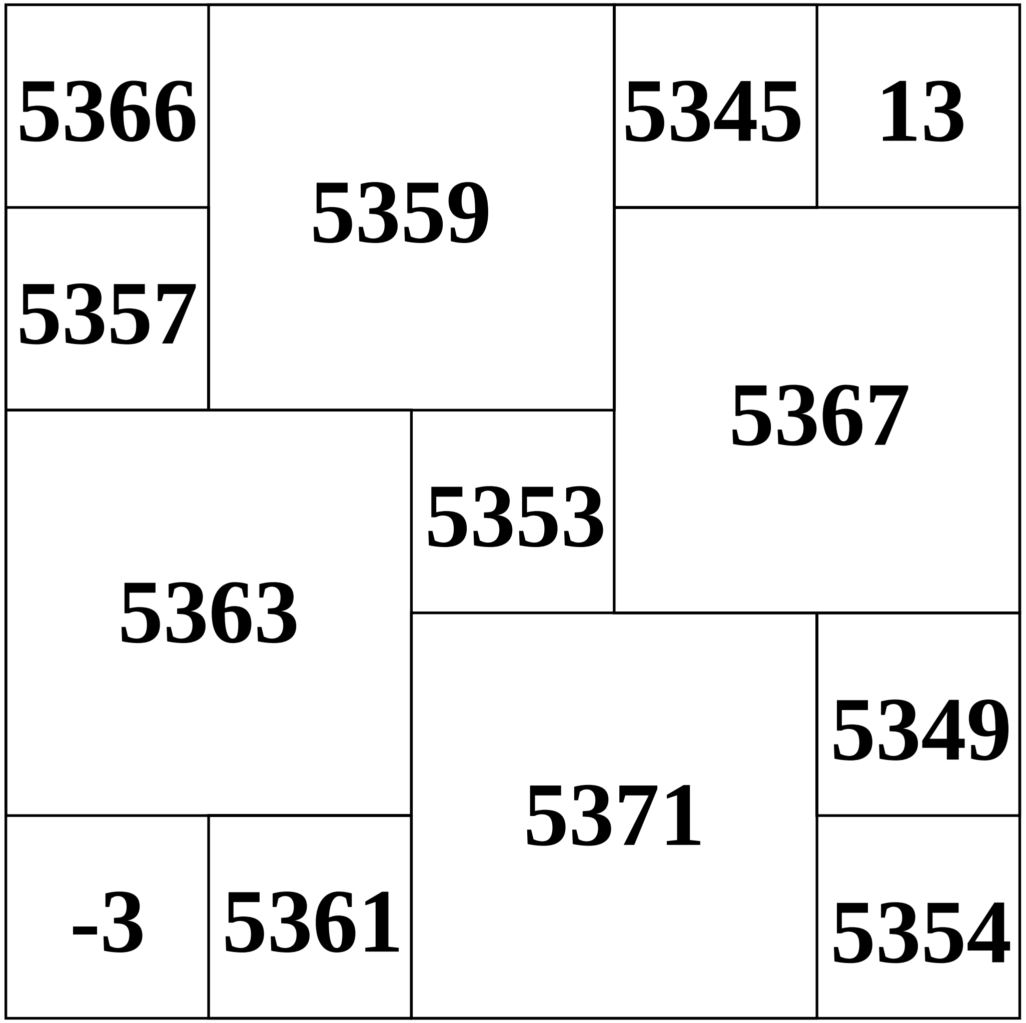


An NSMS of 13 semiprimes



S = 425

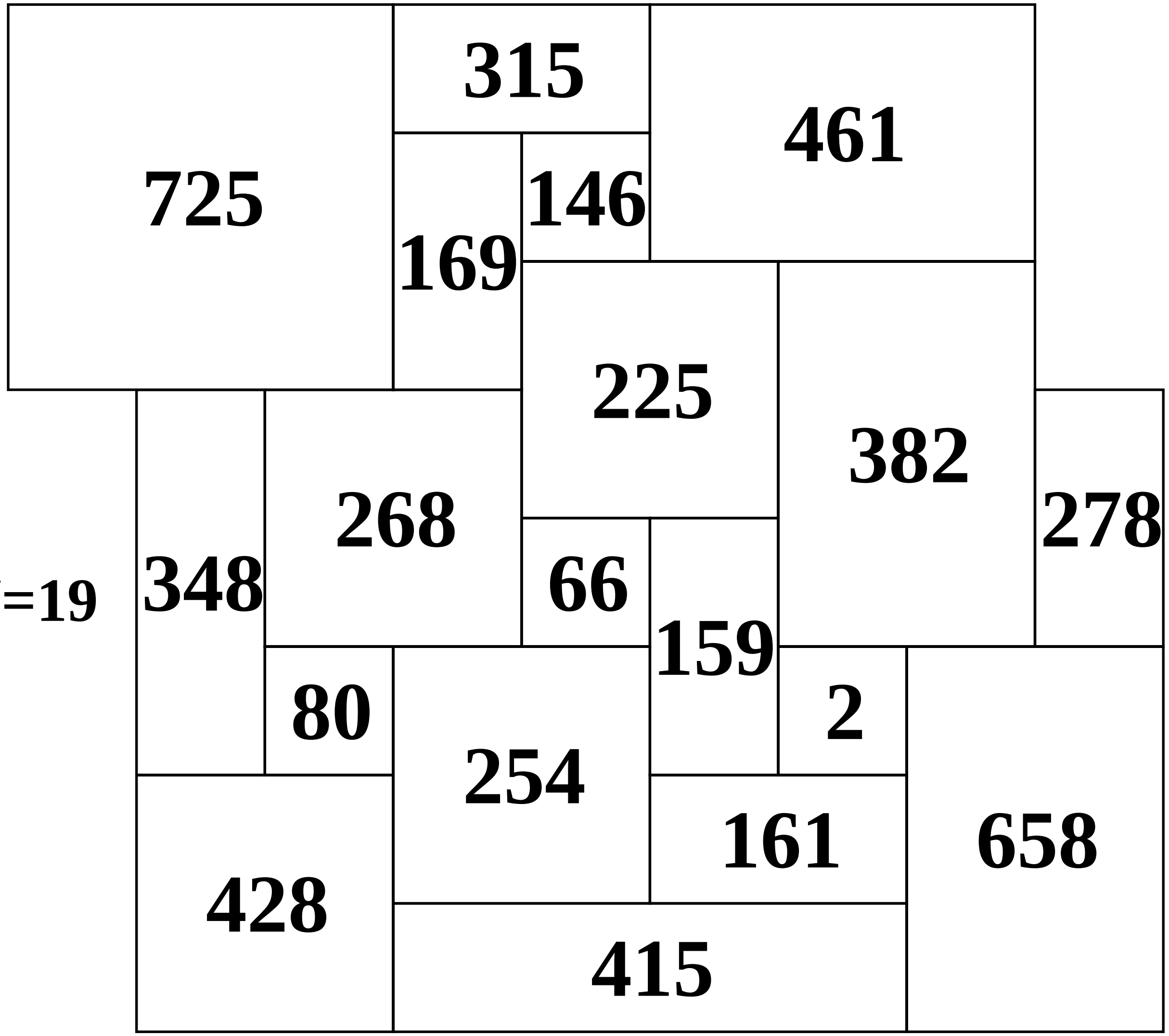
An NSMS with 11 consecutive semiprimes,
and two other numbers, one of which is negative



Contains every semiprime from 5345 to 5371 but also a negative number!

NOT QUITE A NEOSEMIMAGIC CYLINDER!

N=19

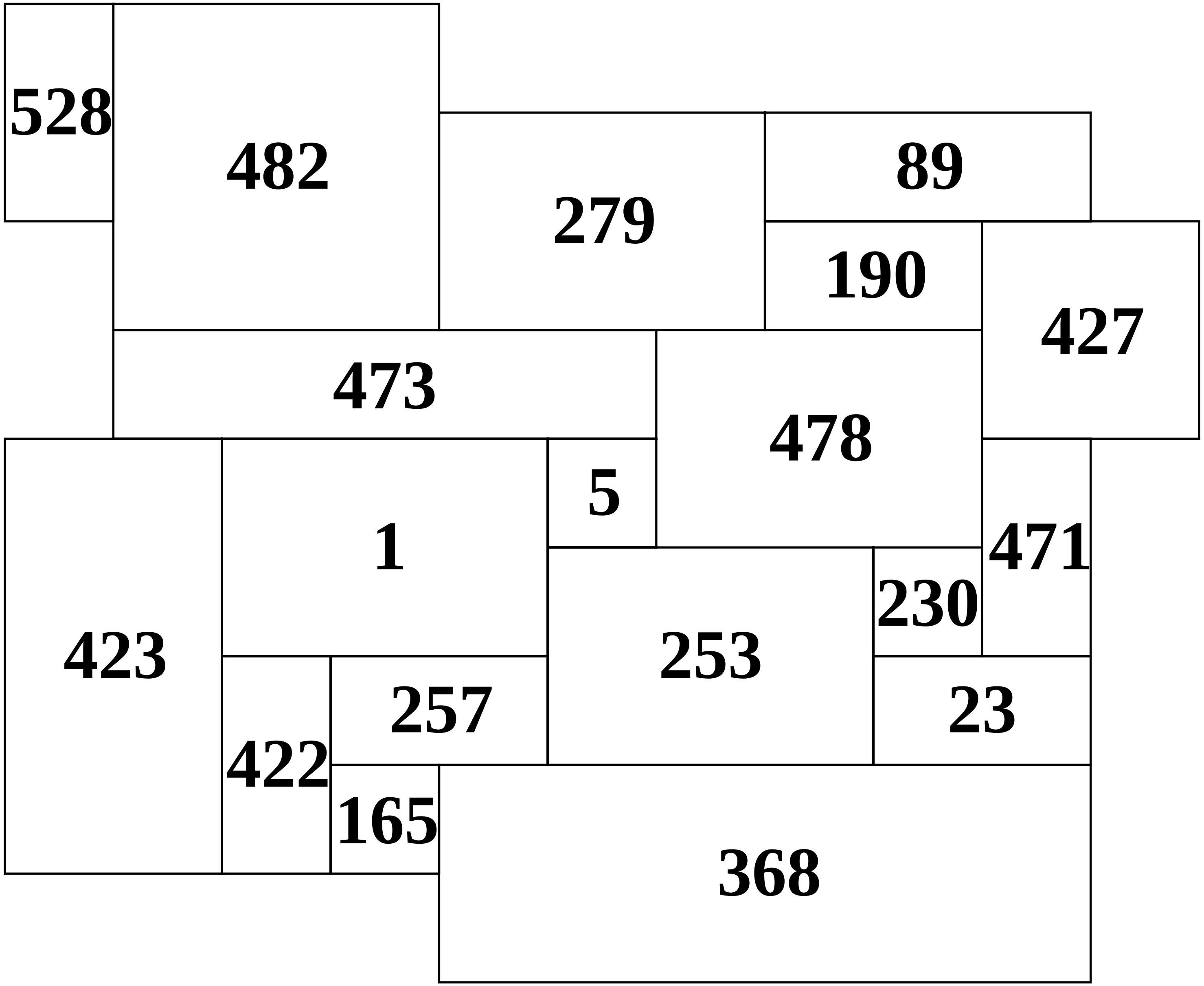


1501 1501 1421 1421 1421 1421 1501 1661 <== Vertical sums

Every horizontal sum is 1501, but the vertical sums have three different values!

NEOSEMIMAGIC TORUS

No problem here. Every horizontal and vertical sum is the same (S=1378).



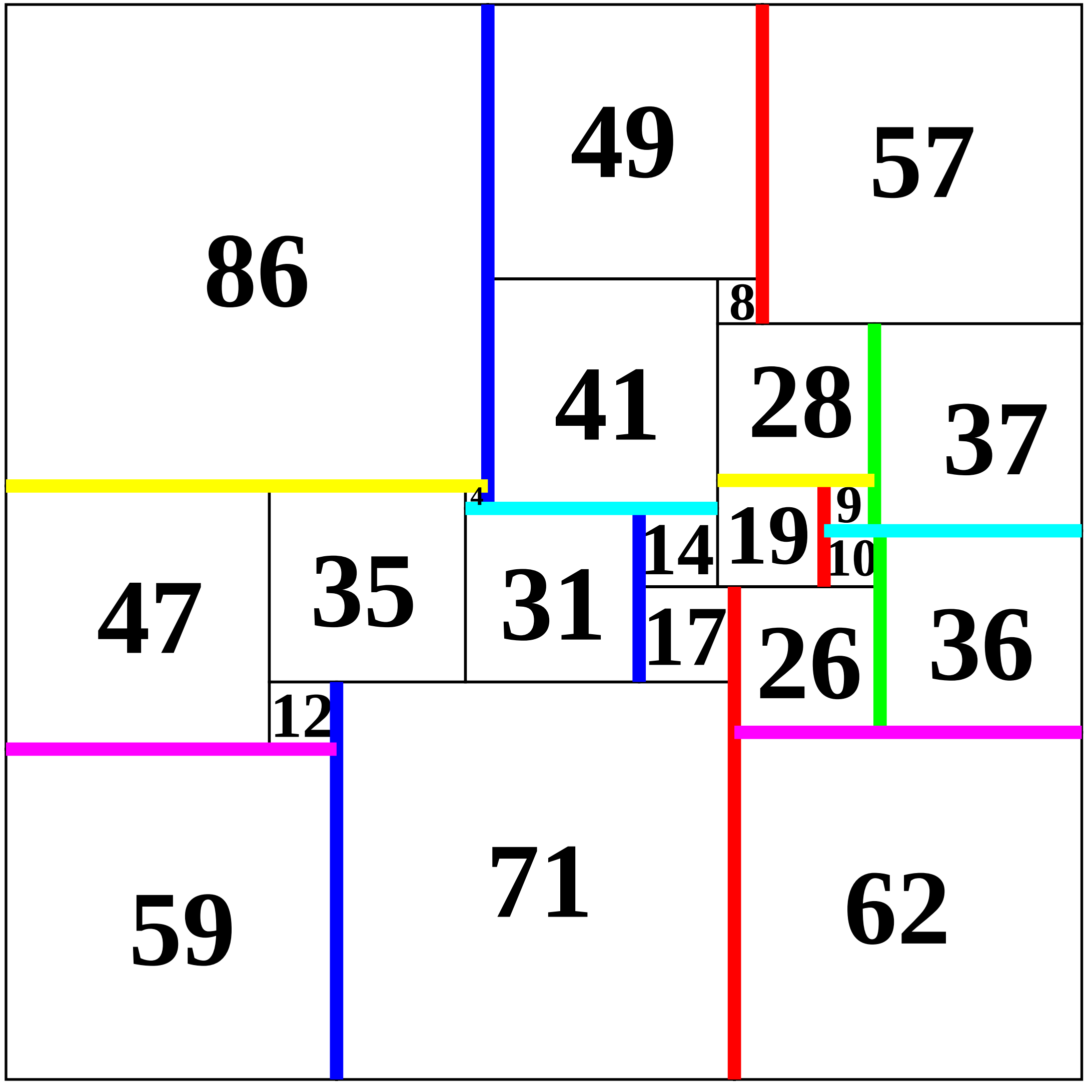
CHALLENGE 2

For each order, what is the smallest magic sum for an NSMR (other than that of a semimagic square if the order is a square)?

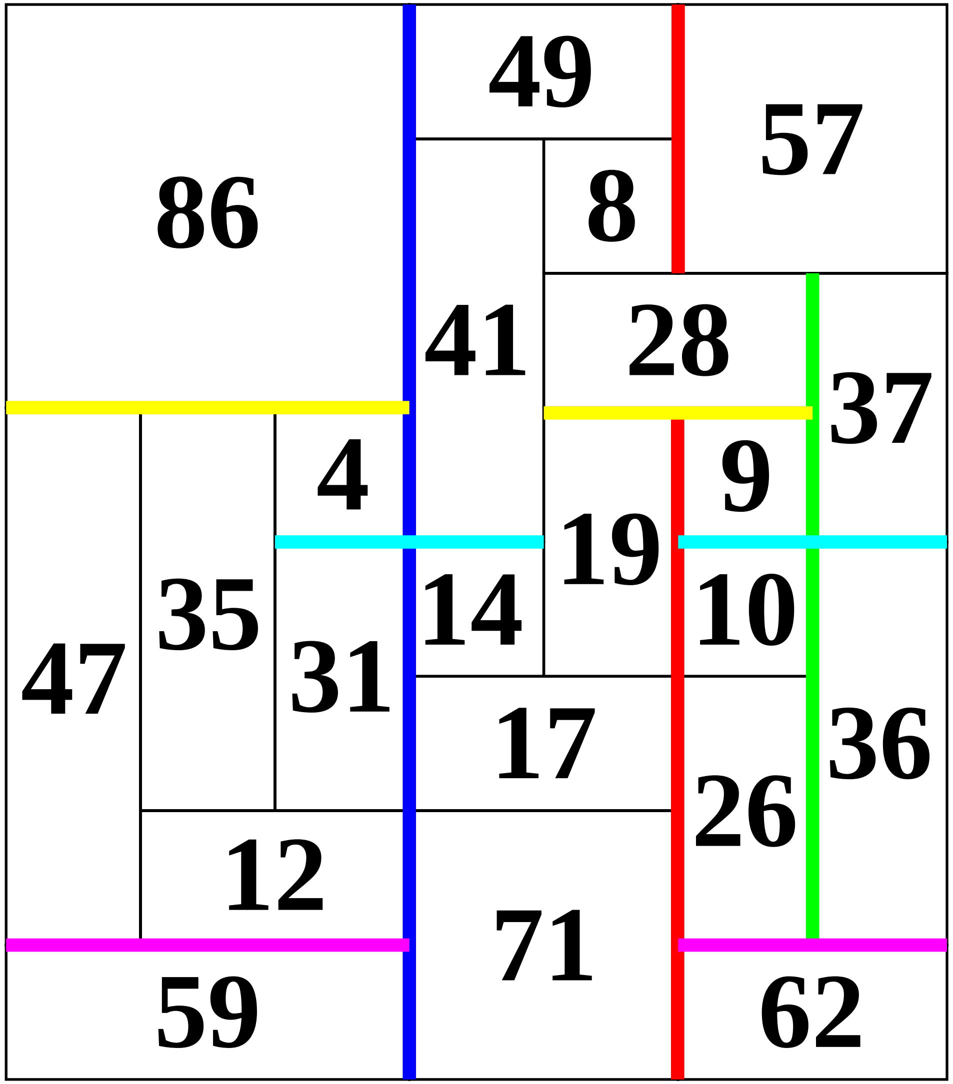
CHALLENGE 3

What if we limit ourselves to NSMRs whose numbers can be replaced by those in a PSS? (See next two slides.)

A PSS/NSMS and an alternate (topologically equivalent) NSMR after certain sets of line segments have been made collinear



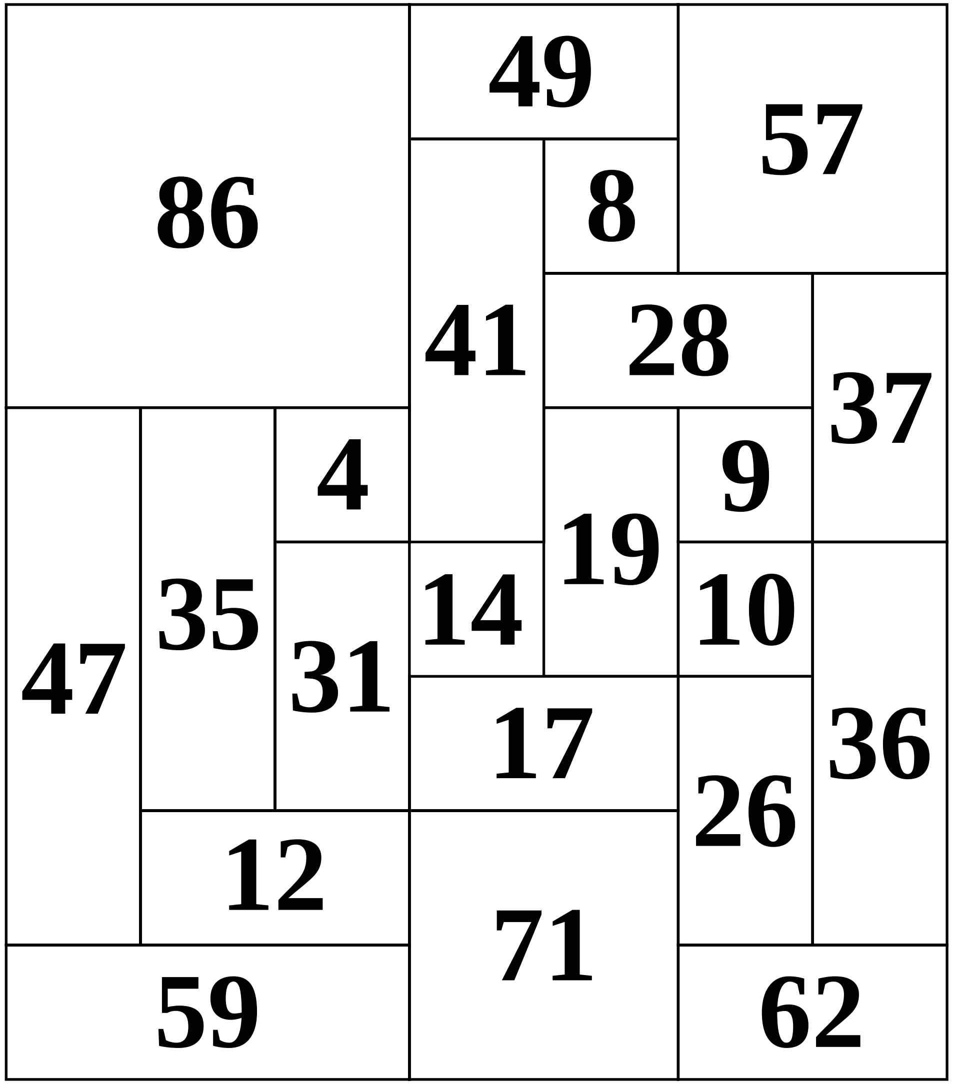
22:192 (AJWD,1978)
The order 22 PSS with the largest side.



An alternate 22:192 NSMR
with the same numbers.

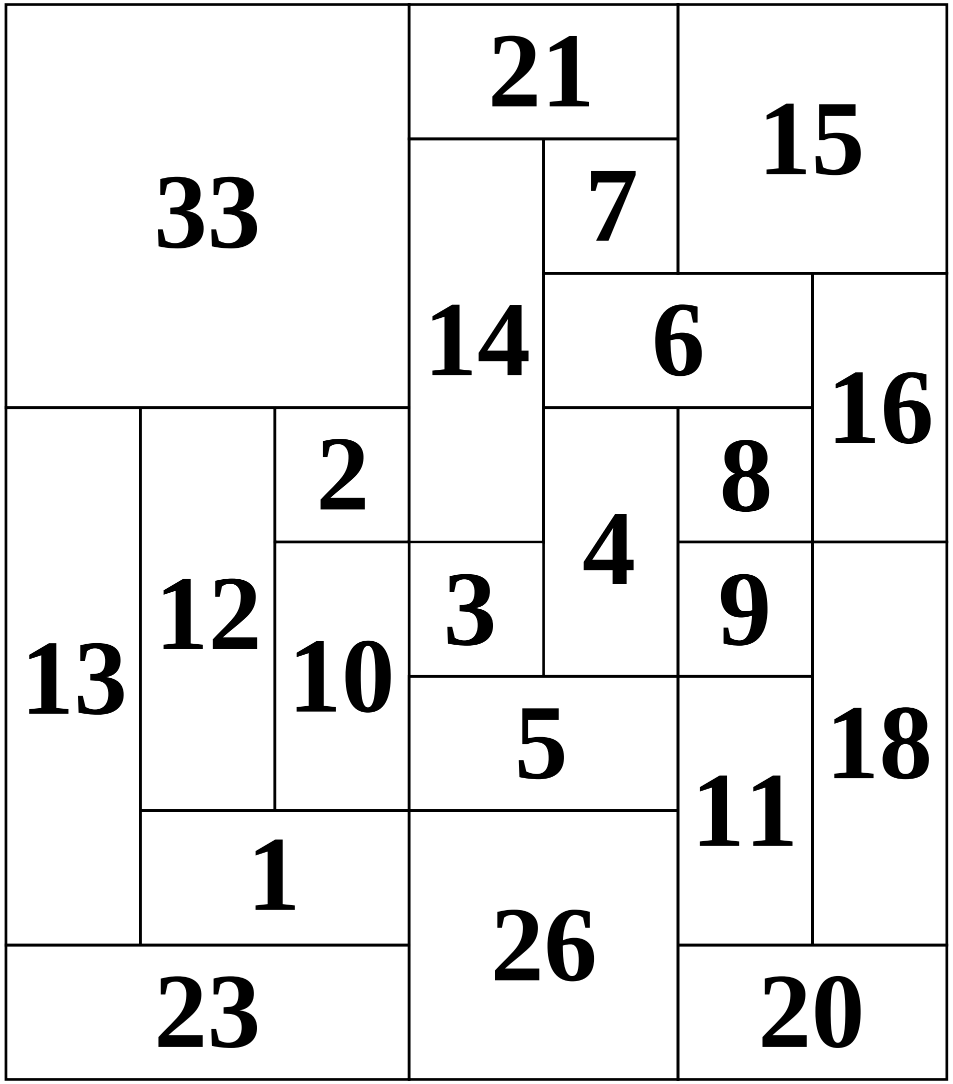
NSMRs with the same tiling with numbers

(A) As in a PSS.



Alternate 22:192 NSMR

(B) With the smallest magic sum.



22:69 (GHM,2018)
Includes the 16 consecutive integers starting with 1.