HOW TO AVOID A HANGOVER USING MATHS

Firstly, although I do come back to the title, I should declare it's just a little intrigue to draw you in.

I just wanted to talk about Prüfer sequences.

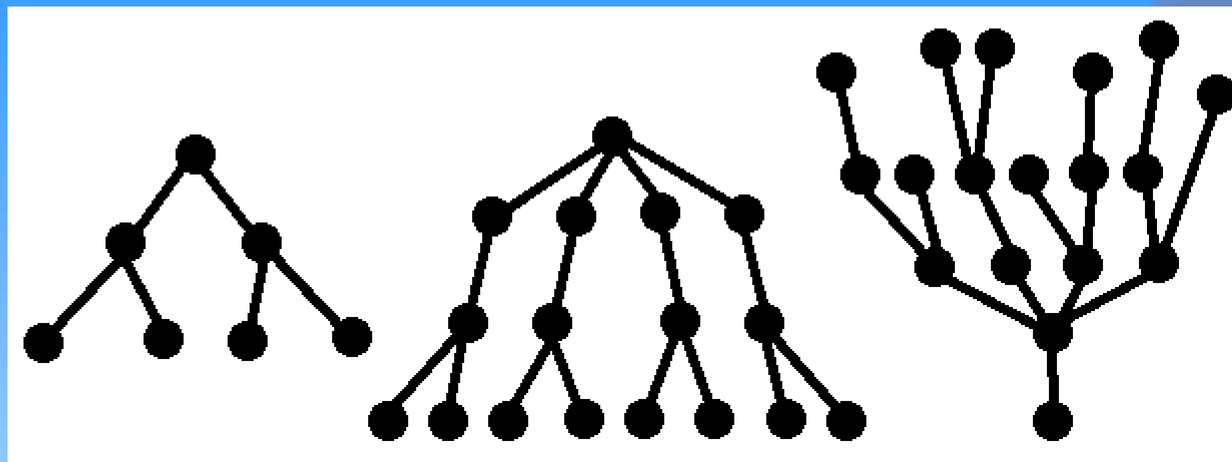
This is because Graph Theory was one on my favourite subjects in maths at university.

They seemed like a neat little thing for a MathsJam talk.

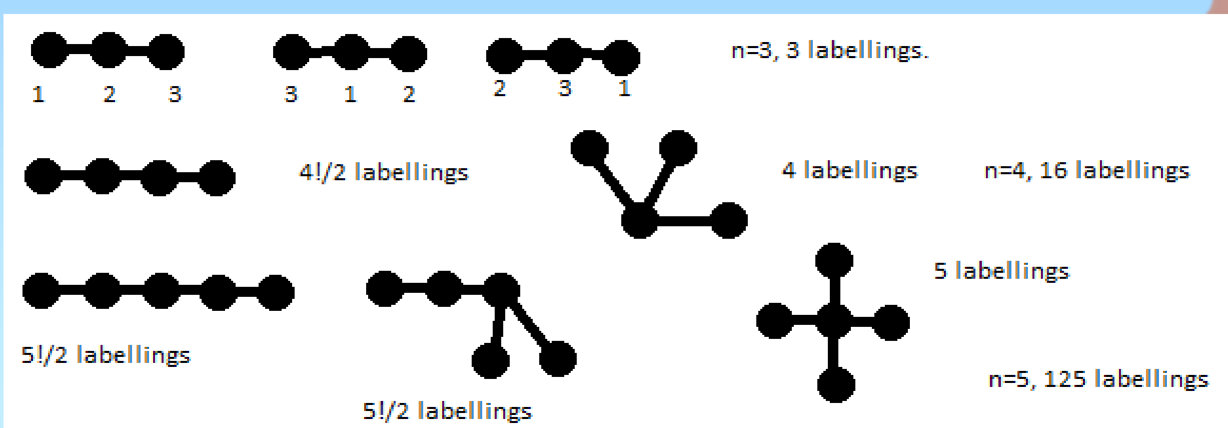
As far as I can remember no-one has talked about them before.

Definitions:

A **tree** is a simple connected graph with no circuits.



A **labelled tree** is a tree with a label attached to each vertex.



<u>Theorem (Cayley, 1889):</u>
There are n^{n-2} labelled trees with n vertices.

There are several proofs of this, but we're going to look at a nice easy one due to Heinz Prüfer, (1896-1934).

Prüfer assigned a sequence to a labelled tree using the following rule:

1. Look at the ‘ends’ and choose the one with the smallest label.
2. Put the label of the adjacent vertex as the first/next, number in the sequence.
3. Remove the end already used and repeat the process until there are two vertices left.

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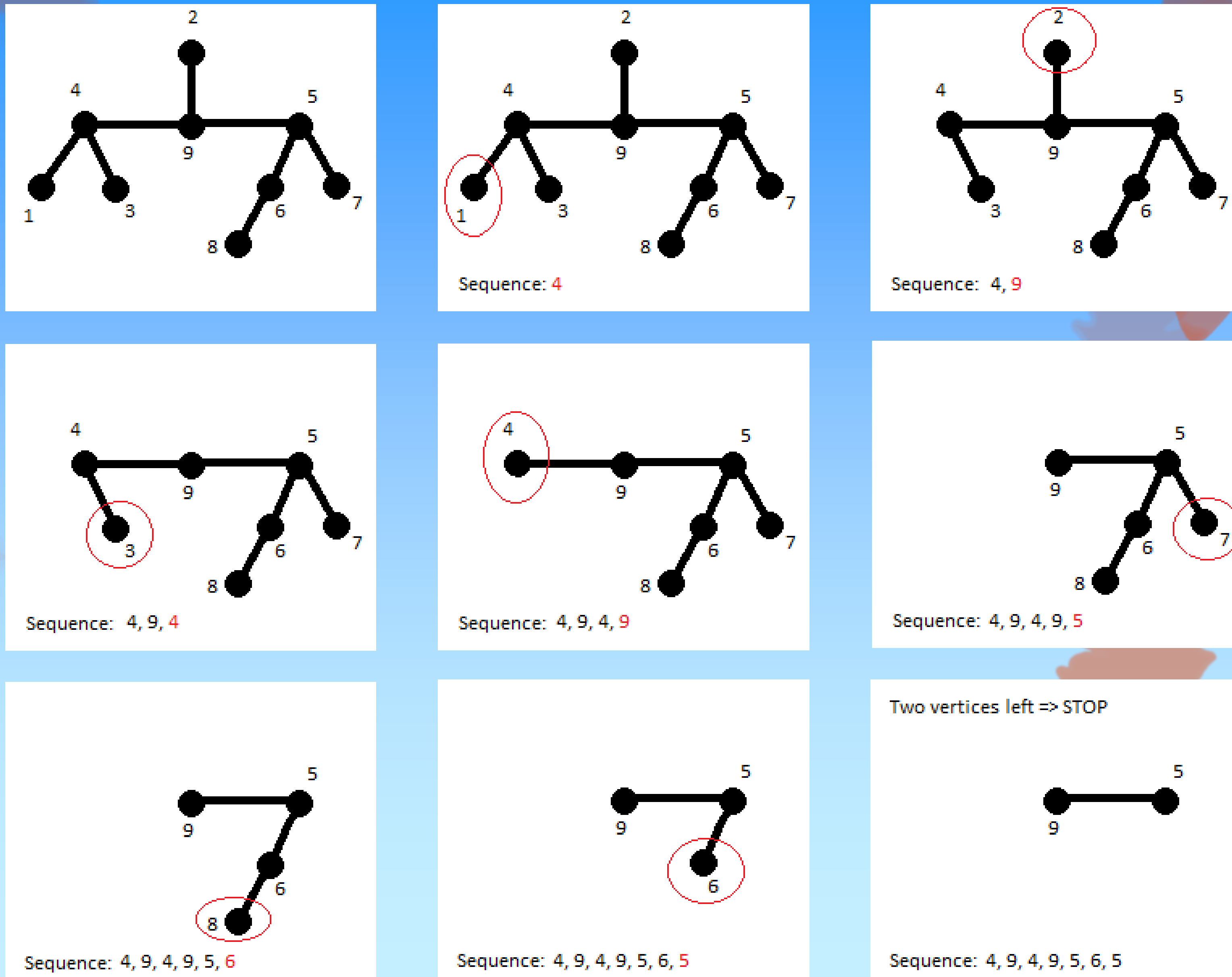

In general, a labelled tree gives a unique sequence, (of length n-2 on the labels 1-n).

Since there are n^{n-2} such sequences, then provided we can show there is a 1-1 correspondence between Prüfer sequences and labelled trees, we will have proven Cayley's theorem.

1. Draw the n vertices and label them 1,2…,n.
2. Write down a list of the labels 1,2…,n. Write down the sequence.
3. Find the smallest number, k, **not** in the Prüfer sequence. Join this vertex to the first element in the sequence.
4. Remove k from the list and delete the first element of the sequence.
5. Repeat from 3. until the list has just two elements. Join them.

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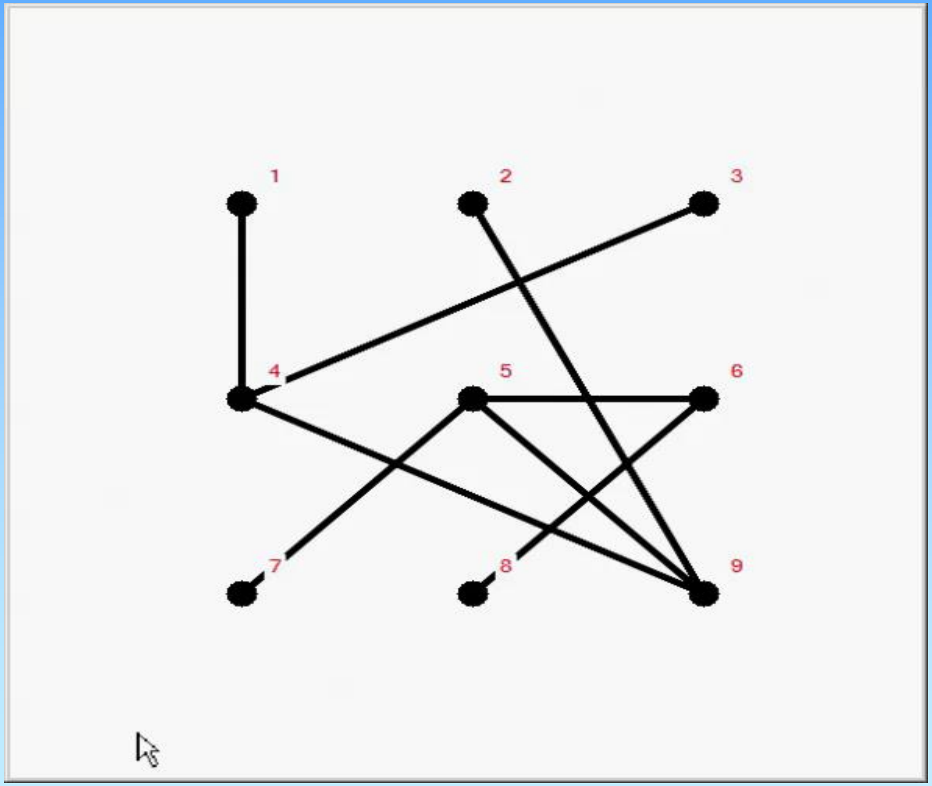
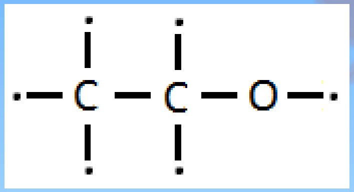


About Hangovers

Some drinks seem to cause worse hangovers than others. E.g. Brandy and red wine, worse than vodka.

Impurities in the alcohol, called "congeners" are thought to be a contributory factor.

(Ethyl) Alcohol is CH_3CH_2OH



It's a saturated hydrocarbon. One of things Cayley was interested in was counting the number of saturated hydrocarbons.

Leave it as an "exercise to the reader" to continue his good work, focussing on the binding sites of the congeners.