Bending vibration of a beam

nodal points and modes

MathsJam Yarnfield Stone November 2019

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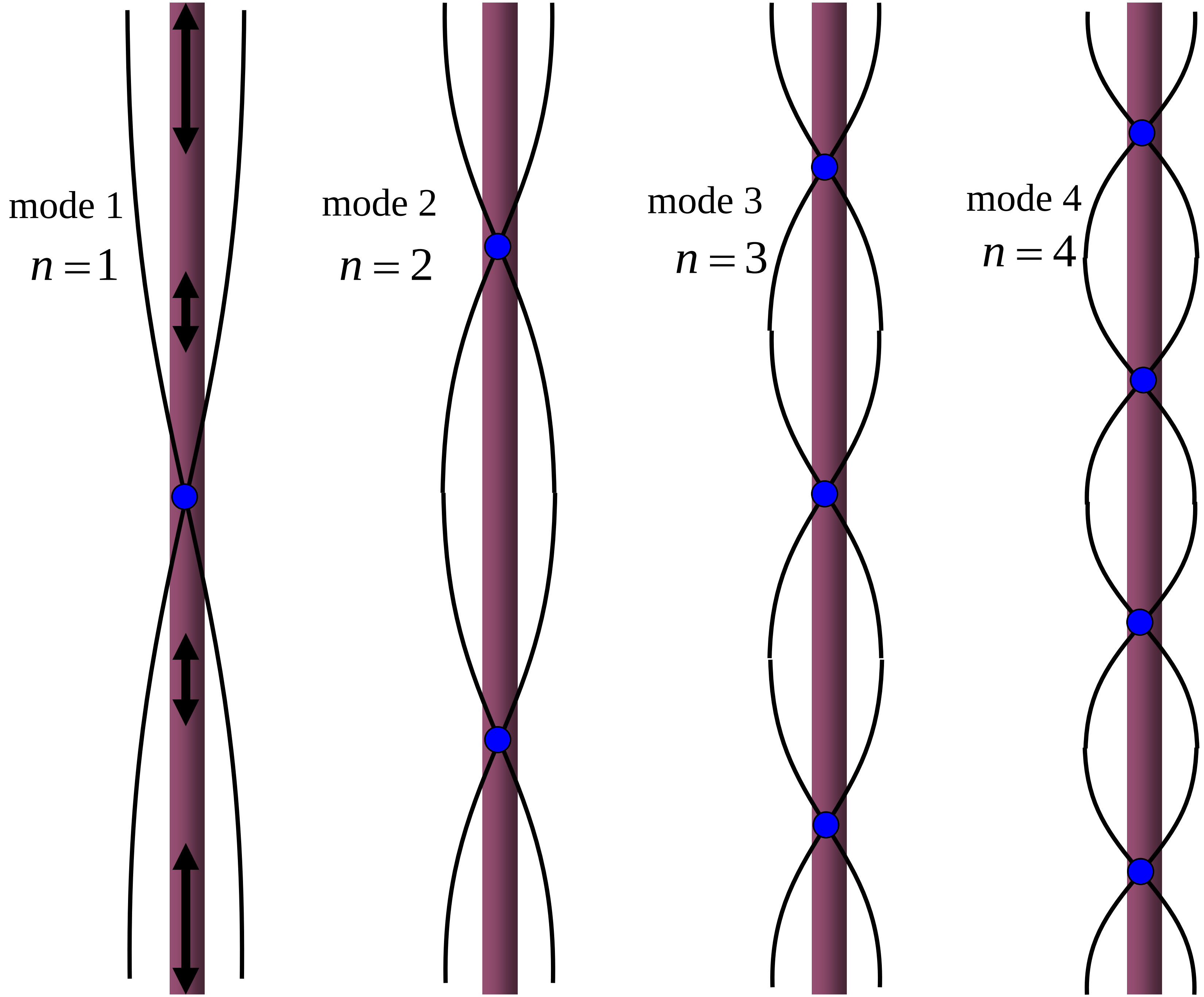
AXIAL VIBRATION

$$f_n = \frac{n}{2L}\ c$$

where $c = \sqrt{\frac{E}{\rho}}$

L = length
ρ = density
E = Young's Modulus



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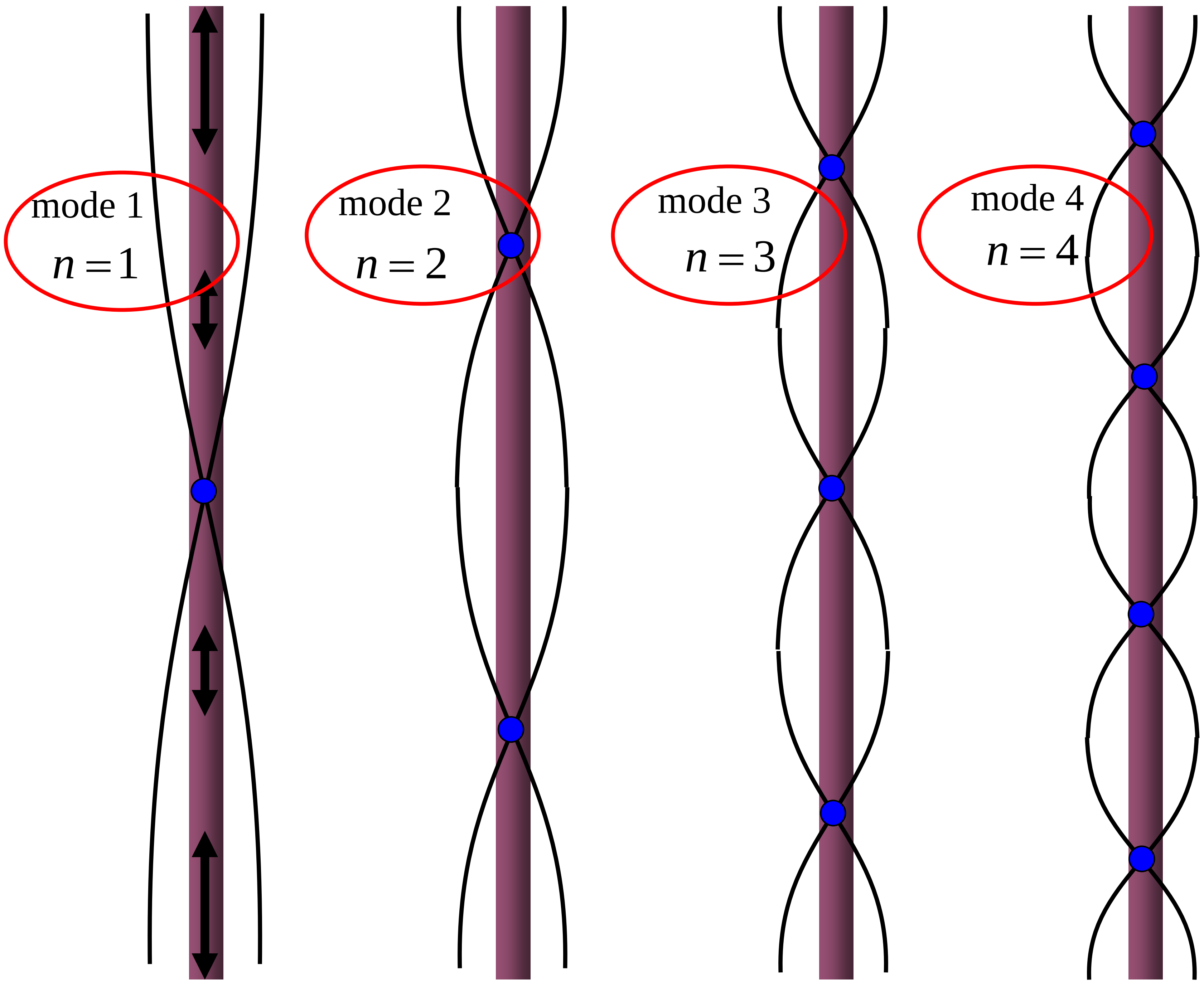
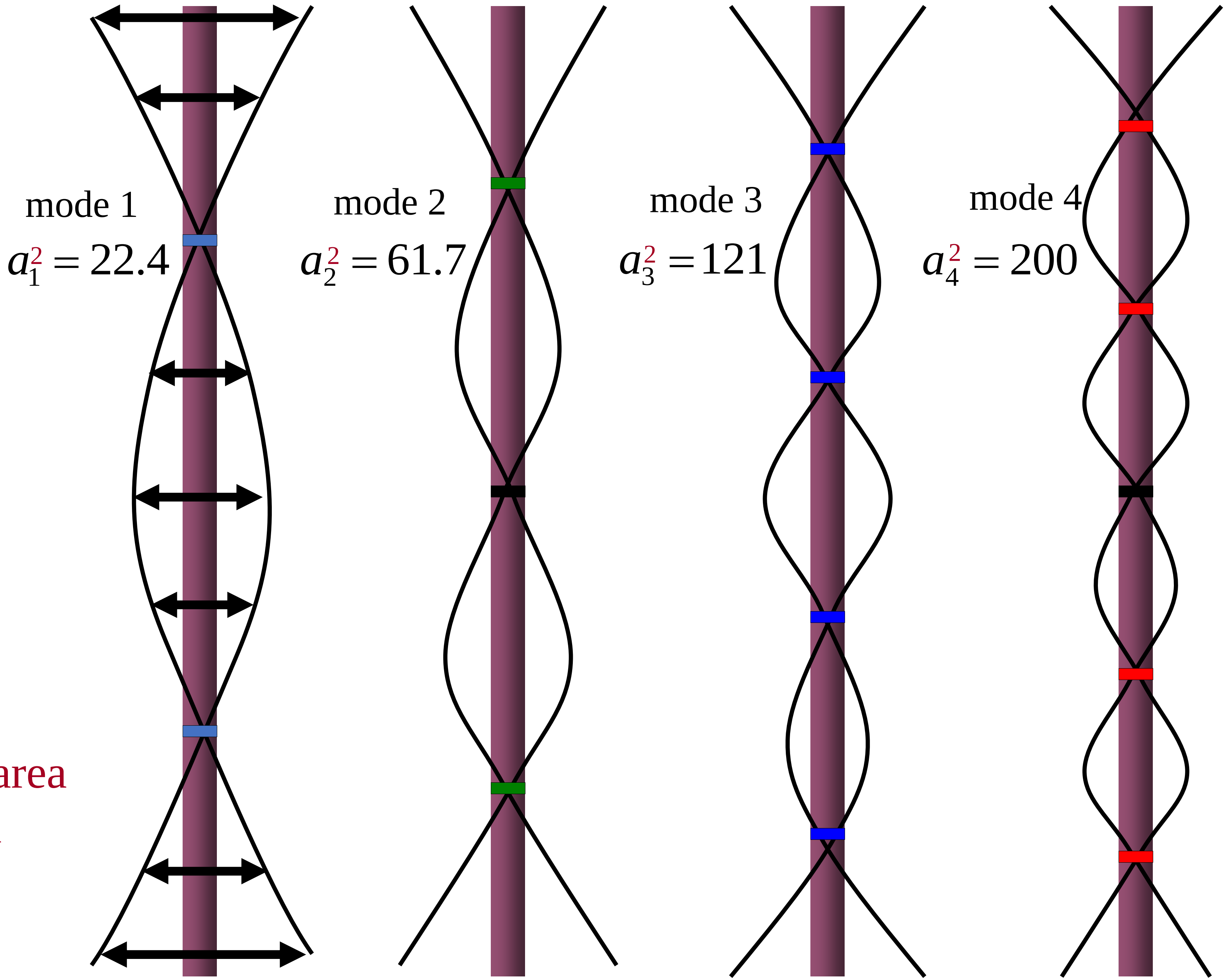


EULER BENDING VIBRATION

$$f_n = \frac{a_n^2}{2\pi L^2}\sqrt{\frac{I}{A}}\;c$$

where $c = \sqrt{\frac{E}{\rho}}$

L = length
I = second moment of area
A = cross-sectional area
ρ = density
E = Young's Modulus



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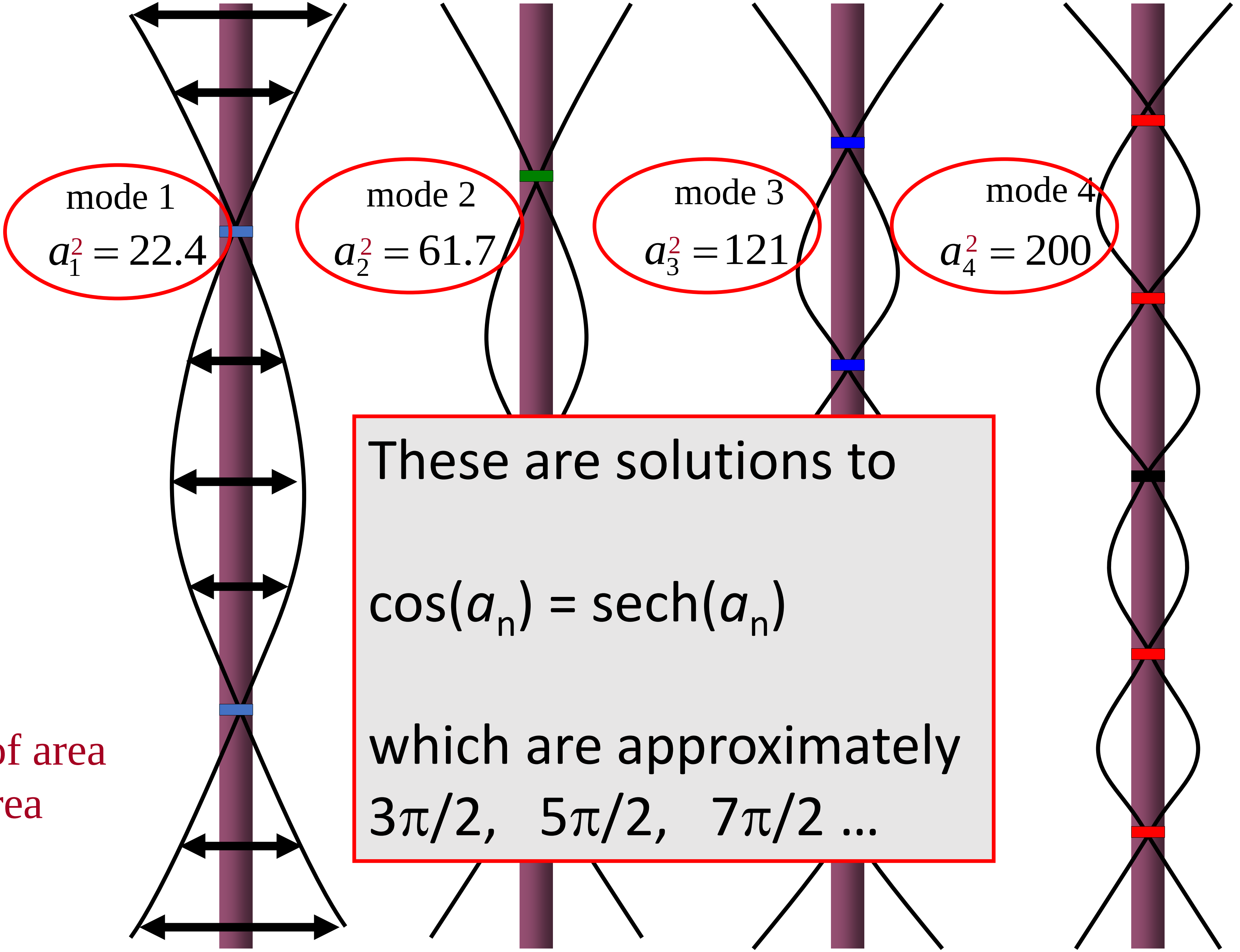


These are solutions to

$\cos(a_n) = \text{sech}(a_n)$

which are approximately
$3\pi/2$, $5\pi/2$, $7\pi/2$ ...

The location of the nodal points for a vibrating beam are:

beam length L=1000mm

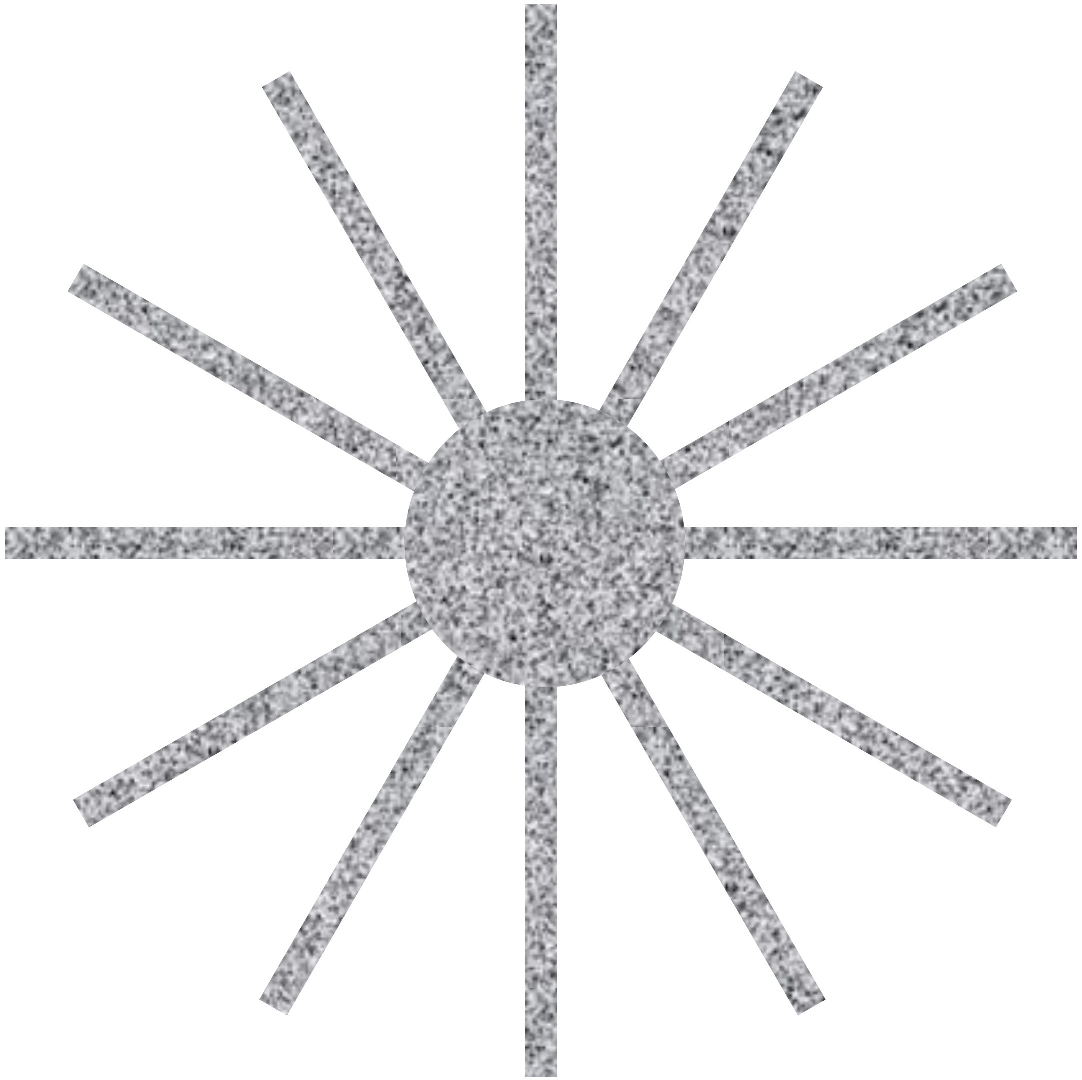
(distance x measured in mm from one end)

mode 1:	224	776							
mode 2:	132	500	868						
mode 3:	94	356	644	906					
mode 4:	73	277	500	723	927				
mode 5:	60	226	409	591	774	940			
mode 6:	51	192	346	500	654	808	949		
mode 7:	44	166	300	433	567	700	834	956	
mode 8:	39	147	265	382	500	618	735	853	961

These are zeros of

$$u = (\cos(a_n L) - \cosh(a_n L))\,(\cos(a_n z) + \cosh(a_n z)) \quad + (\sin(a_n L) + \sinh(a_n L))\,(\sin(a_n z) + \sinh(a_n z))$$

with $z = x/L$



Axisymmetric bodies

Turbocharger blade vibration

Very weird behaviour!