Is (one of the) biggest always best?

Adam Atkinson (ELHP)
MathsJam Gathering, 2021

Is (one of the) biggest always best?

Abstract: No

Adam Atkinson (ELHP)
MathsJam Gathering, 2021


SMART GAMES


PROVACI

Not an open day problem

I give you a heap of *n* tokens. (*n* is a non-negative integer).

Not an open day problem

I give you a heap of *n* tokens. (*n* is a non-negative integer).

You split it into as many sub-heaps as you wish, but their sizes must be distinct prime numbers.

Not an open day problem

I give you a heap of *n* tokens. (*n* is a non-negative integer).

You split it into as many sub-heaps as you wish, but their sizes must be distinct prime numbers.

I multiply all their sizes together and pay you the product in GBP.

Not an open day problem

How do you maximise the amount I have to pay you?

Some numbers can’t be done

1, 4 and 6 are impossible.

Are there any others? No.

0 *is* possible.

It’s a sum of 0 primes with product 1.

11

Split	How many primes?	What is their product?
11	1	**11**

10

Split	How many primes?	What is their product?
7 3	2	21
5 3 2	3	**30**

10

Split	bigosity	bestitude
7 3	2	21
5 3 2	3	**30**

20

Split	bigosity	bestitude
17 3	2	51
13 7	2	91
13 5 2	3	130
11 7 2	3	**154**

30

Split	bigosity	bestitude
23 7	2	161
23 5 2	3	230
19 11	2	209
17 13	2	221
17 11 2	3	374
13 7 5 3 2	5	**2730**

40

Split	bigosity	bestitude
37 3	2	111
31 7 2	3	434
29 11	2	319
23 17	2	391
23 7 5 3 2	5	4830
19 13 5 3	4	3705
19 11 7 3	4	4389
19 11 5 3 2	5	6270
17 13 7 3	4	4641
17 13 5 3 2	5	**6630**

Does this keep happening?

No.

It fails for the first time for n=319

Looking for examples

n	$\log_e$(max product)	Best split	Biggest split
319	38.3	13	14
372	42.3	14	15
492	50.5	16	17
703	63.2	19	20
865	72.0	21	22
954	76.5	22	23
1584	104	28	29
1842	114	30	31
2112	123	32	33

The first quite a few: A053020

319 372 492 703 865 954 1584 1842 2112 2118 2418 2569 2575
2899 2905 3078 3432 3438 4212 4218 4423 4429 5341 5815 5821
6066 6072 6323 6329 6592 7132 7967 7973 8254 8260 8266 9502
9508 9514 9839 9845 10176 10182 11225 11231 11590 11598
11951 11957 12324 12330 13086 13092 13481 13866 13890
14279 14682 14688 15522 15528 16392 17268 17274 17723
19565 19571 20050 21028 22024 22030 22527 22533 22539
23044 23074 23571 23577 23583 24118 24124 24140 24665
24671 24681 25222 25228 25791 26931 27502 27508 27518
28089 28095 28688

Questions

Do these examples continue forever?

Can the best split be 2 smaller than the biggest?

Can the best split be *k* smaller than the biggest?