Speaking Truth to Powers

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MathsJam Gathering, November 2023

"In 1937 A.D., a German-Jewish mathematician named Samuel Isaac Krieger, who was taking a mineral bath near Buffalo, N. Y., suddenly leaped out, rushed naked into the adjoining room, began to scribble figures. He thought he had discovered something" *TIME Magazine, March 1938*

Lincoln Baths in Saratoga Spings, near Buffalo, New York. (Above: c.1940, below: 2008). Where Krieger had his famous bath?

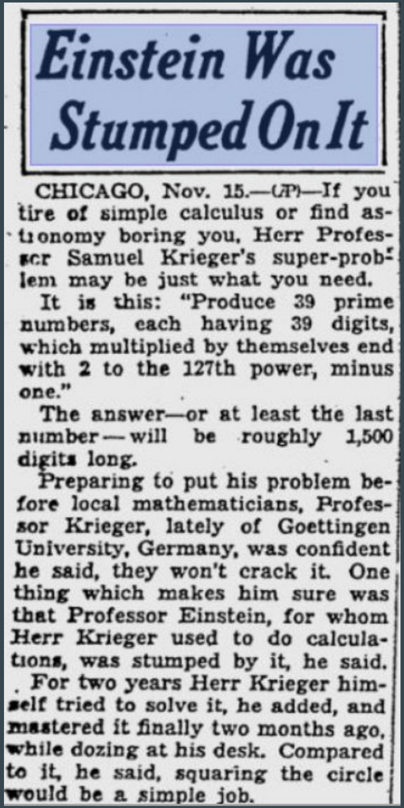
“Greatest genius in mathematics”

“Dr. Samuel Isaac Krieger, ... flunked three times in arithmetic, back in preparatory school in Hamburg, Germany, but he didn’t let a little thing like that stop him from becoming a great mathematician.

He has been described as “the greatest genius in mathematics and the greatest mathematical mind I have ever seen” by no less a personage than famed Dr. Albert Einstein, relativity expert who is said to be somewhat of a mathematician himself and well aware that the answer to two plus two is four.” *Milwaukee Journal, 1935*

“In Chicago, one Samuel I. Krieger covered 72 sheets of foolscap paper, wore six pencils to stubs ... announced a 72-digit prime number, challenged the world to show him a bigger one.

His number:
231,584,178,474,632,390,847,141,970,017,375,815,706,539,969,331,281,128,078,915,826,259,279,871.”
TIME Magazine Science column, December 1935



Einstein Was Stumped On It

CHICAGO, Nov. 15.—(AP)—If you tire of simple calculus or find astronomy boring you, Herr Professor Samuel Krieger's super-problem may be just what you need.

It is this: “Produce 39 prime numbers, each having 39 digits, which multiplied by themselves end with 2 to the 127th power, minus one.”

The answer—or at least the last number—will be roughly 1,500 digits long.

Preparing to put his problem before local mathematicians, Professor Krieger, lately of Goettingen University, Germany, was confident he said, they won't crack it. One thing which makes him sure was that Professor Einstein, for whom Herr Krieger used to do calculations, was stumped by it, he said.

For two years Herr Krieger himself tried to solve it, he added, and mastered it finally two months ago, while dozing at his desk. Compared to it, he said, squaring the circle would be a simple job.

Pittsburgh Post-Gazette November 16, 1933

Krieger’s discovery

Krieger's discovery

(The integer $2 < n \leq 20$ was withheld)

$$1324^n + 731^n = 1961^n$$

Fermat's Great Conjecture

$1324^n + 731^n = 1961^n$

For positive integers x,y,z, the equation $x^n+y^n=z^n$ has no solutions for $n > 2$.

- $n = 4$: proved by Fermat, who also showed you only need to consider primes

$$x^{mp} + y^{mp} = z^{mp} \text{ is true if and only if: } (x^m)^p + (y^m)^p = (z^m)^p$$

- (1637–1839) Proved for n = 3, 5, 7
- (~1815–1831) Sophie Germain showed that any counterexamples to Fermat's theorem for primes $p > 5$ must be numbers "whose size frightens the imagination", around 40 digits long.
- (1994) Fully proved by Andrew Wiles.

Fermat's Great Conjecture $1324^n + 731^n = 1961^n$

- "An astute reporter from the New York Times, no baby in mathematics himself, pored over this equation. The reporter saw that
  - the first number raised to any power at all would end in either 6 or 4
  - the second raised to any power would end in 1,
  - and the third raised to any power would end in 1."
- $4^n + 1 = (4 + 1)$ or $(6 + 1)$

  $\neq 1 \pmod{10}$
- "You mean," said Krieger, "that you doubt me?" The reporter admitted it.
  "Well," said Samuel Isaac Krieger sadly, "when the time comes, I will explain everything."

NB. The "Krieger prime":
231584178474632390847141970017375815706539969331281128078915826259279871 = 47 ×
4927322946268774273343446170582464163968935517686832512317358005516593

How far can we push the observation?

Fix n (eg. pick n = 11)

Can we find a number m such that:

$x^n + y^n \equiv z^n$ (mod m) has no solutions at all?

(ignoring some trivial ones, like $m^n + m^n \equiv m^n$ mod m)

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Sadly not, by a theorem of Schur:

Fix an n > 2.

There will always be a prime large enough, such that:

$x^n + y^n \equiv z^n$ (mod p)

has a non-trivial solution, ie. with $xyz \not\equiv 0$ (mod p).

The result is essentially a corollary to a Ramsey colouring theory.